Risk Analysis of complex systems using Bayesian Networks

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Move Forward with Confidence

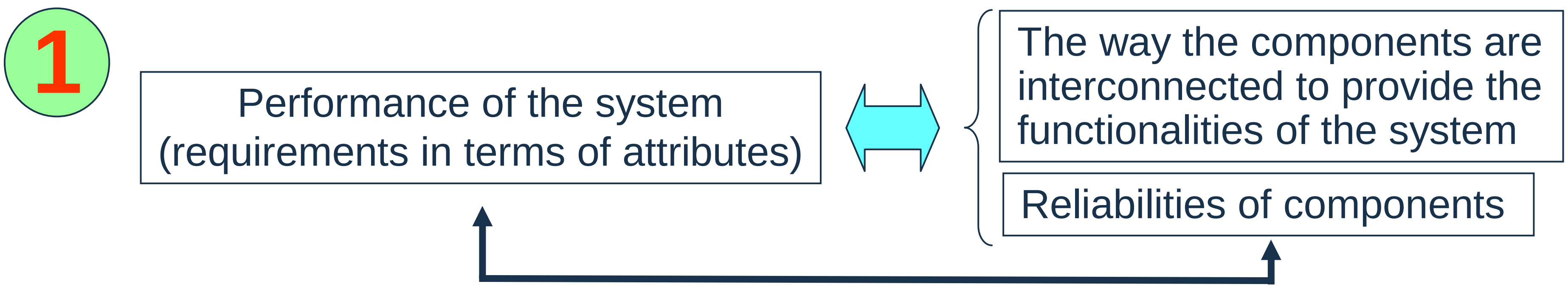
Introduction (1)

Engineered systems such as:

- electricity / water distributions systems
- structural systems

are **complex systems** in the sense that

- ❑ they include geographically distributed and/or functionally **interrelated components**
- ❑ which through their connections with other components provide the desired **functionality of the system** expressed in terms of one or more attributes
- ❑ Different levels of analyses provided by different experts are required



Introduction (2)

2 ***FMECA : Failure Mode, Effects and Criticality Analysis***

Component level (component failure modes) → System level (system failure modes) → Consequences → Personnel / Environment / Asset

3

Component level

System level

Optimisation of the target reliabilities of components ←

→ Checks of performance of the system

4

Component approach is relevant because:

- in general, what are effectively designed are components
- maintenance of a system usually addresses the components of the system

but:

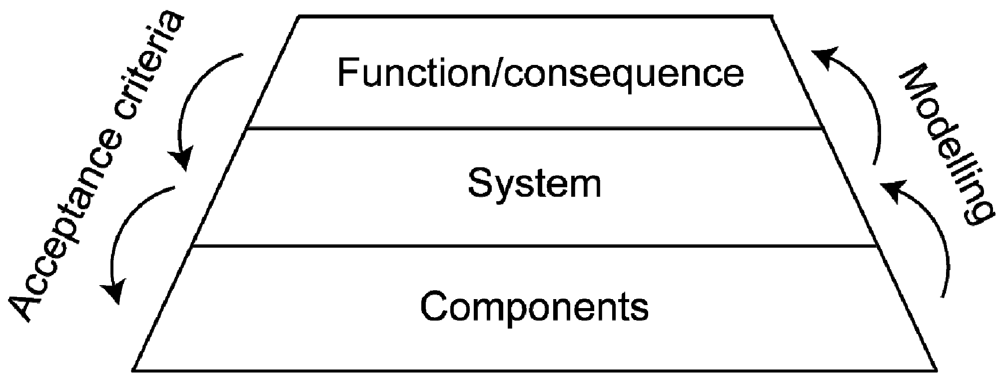
- system performance is the direct concern

Introduction (3)

- Need of a **hierarchical modelling of complex engineered systems** which integrates the different levels of analysis

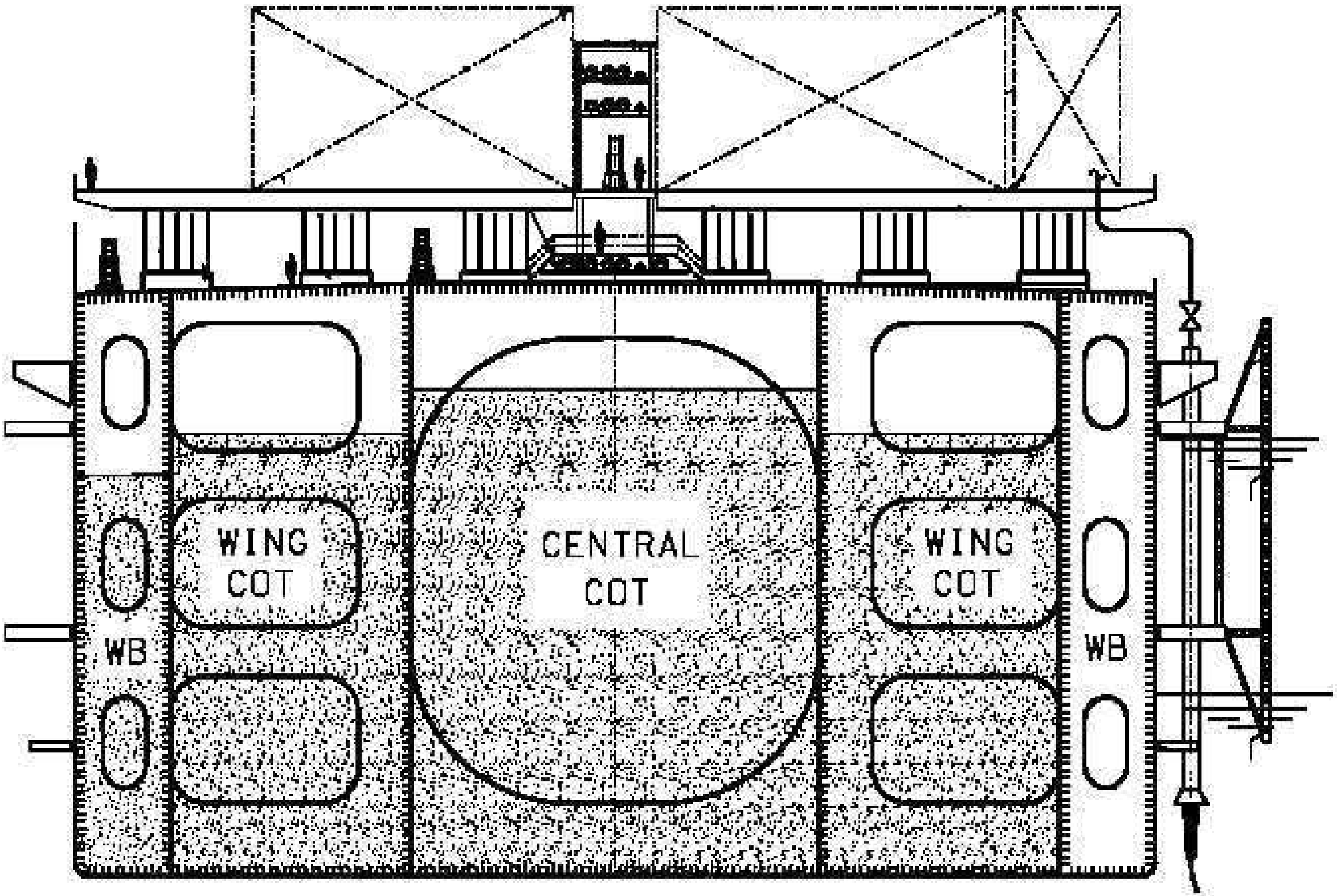
- If performances of the system are expressed in terms of acceptable risks (Personnel, Environment, Asset), then the hierarchical model can be used for **risk assessment** of complex systems

- Example is a **FPSO** (**F**loating **P**roduction **S**torage and **O**ffloading) unit which constitutes a typical complex system. In the examples the ship hull structure is modelled and analysed as a system of sub-systems and risk evaluation of the hull is performed with regard to fatigue deterioration of welded connections and corrosion.

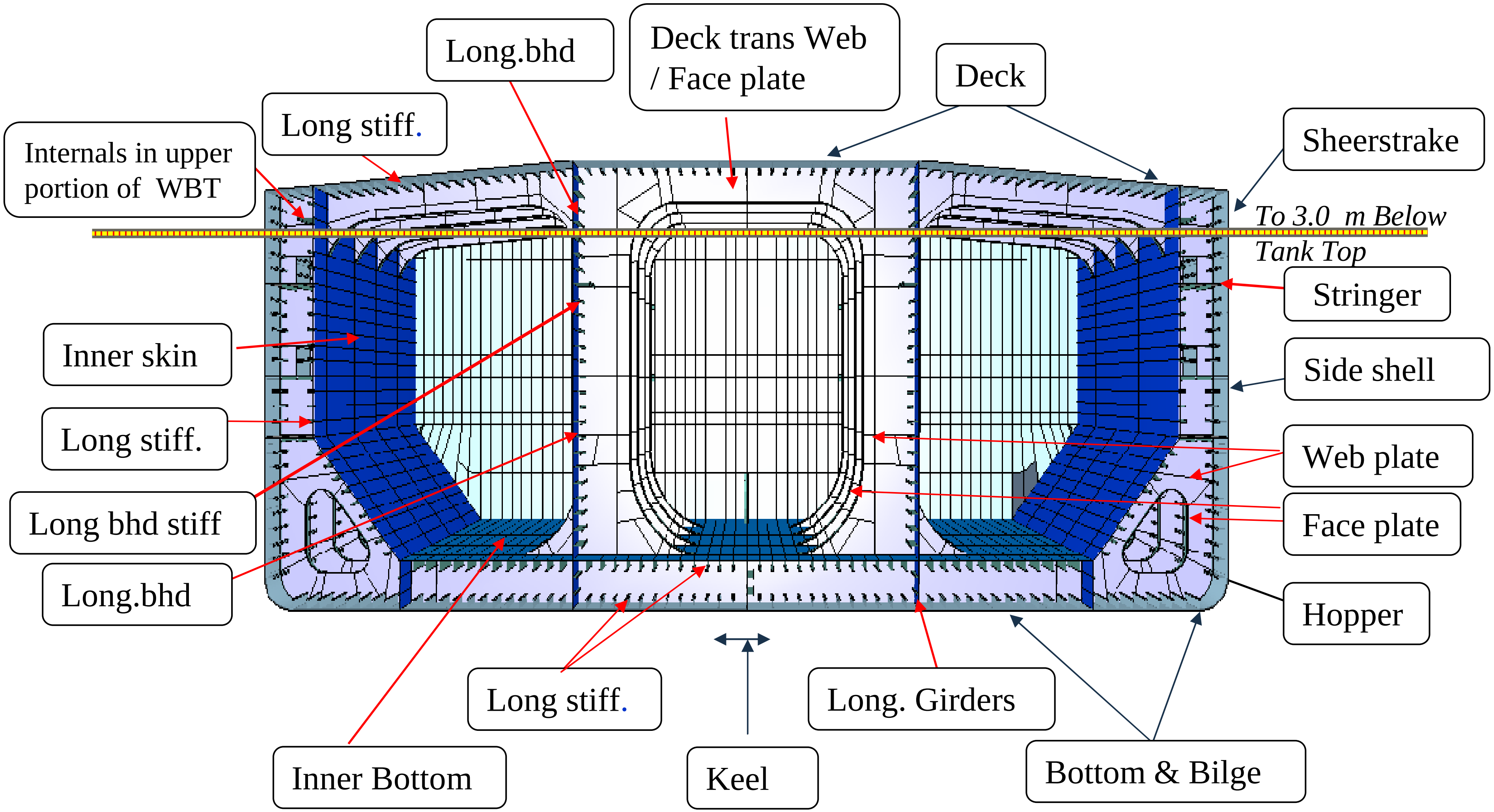


Examples of FPSO

FPSO Structure: General Arrangement – Typical section

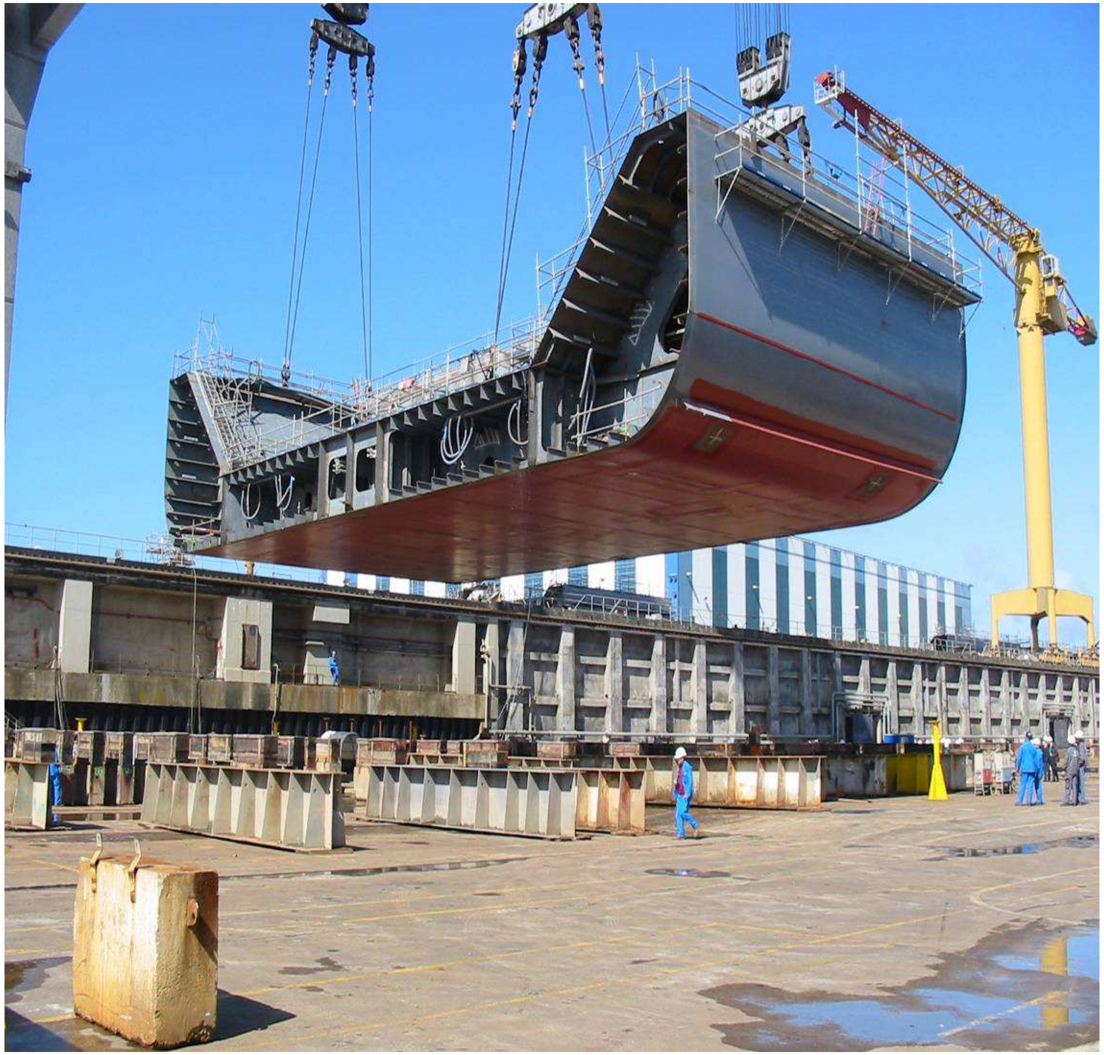


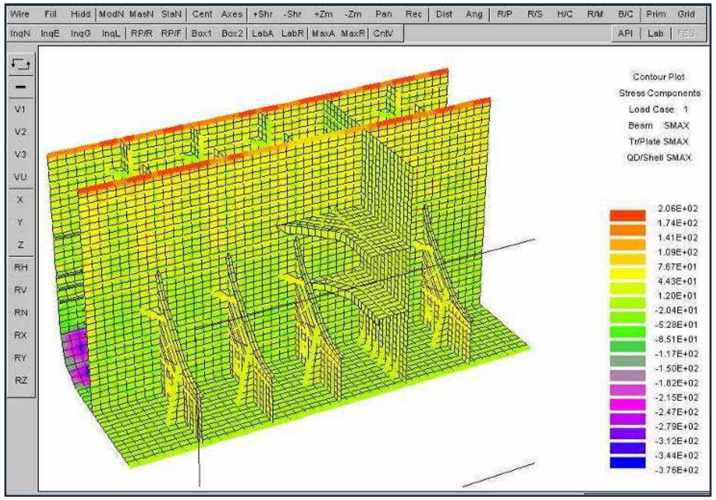
FPSO Hull Structure : typical components



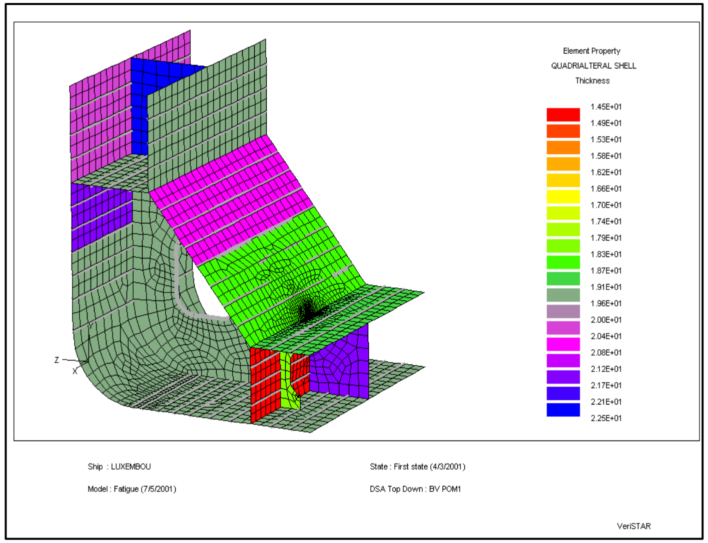
FPSO Hull Structure : Shipbuilding and engineered calculations

Shipbuilding



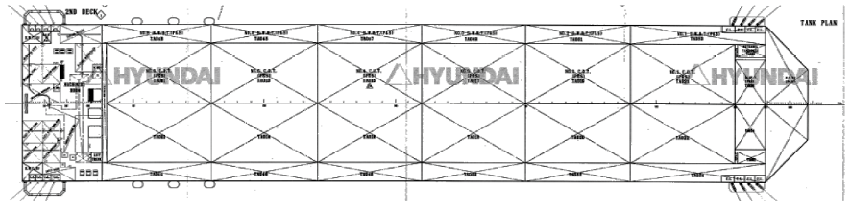


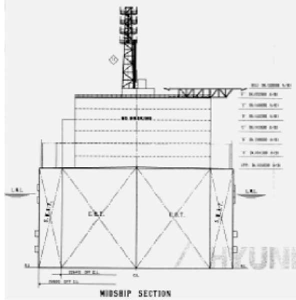
FEM model from VeriSTAR Hull



Fatigue life analysis of hopper knuckles

Hull Structure analysed as a set of interrelated components / sub-structures





- components/(sub-)structures are interrelated:
  - Stiffeners, plates, welded joints
  - Stiffened panels, boundaries
  - Tanks
- different levels of analyses are required such as:
  - Yielding, buckling, fatigue, fracture and corrosion of materials / components
  - System structural analysis
  - Consequence analysis in terms of Personnel, Environment and Asset.

Hull analysed at component level or system level

Method 1 (component level)

Method 2 (system level)

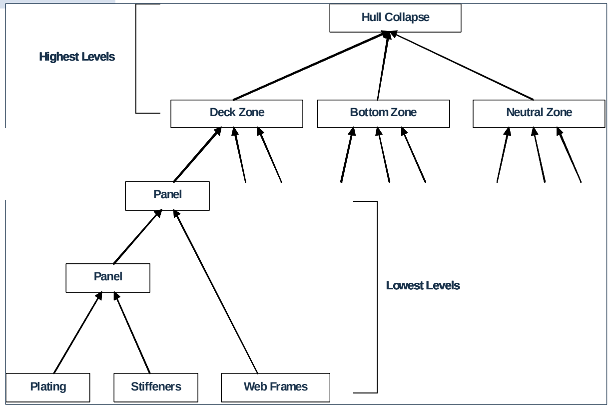
Given RAC at the system level

RAC allocation

RAC at the component level (maximum value of P_f)

Comparison

Comparison

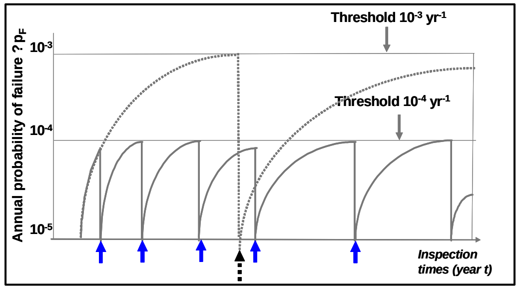


Risk indicator calculated at the system level

Expansion via Hierarchical model

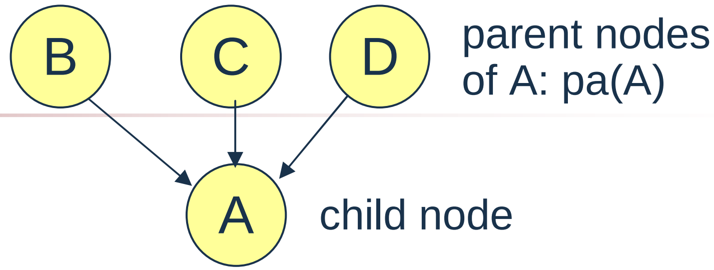
2 approaches

- Constrained reliabilities
- Unconstrained reliabilities



P_f calculated at the component level

Bayesian Networks

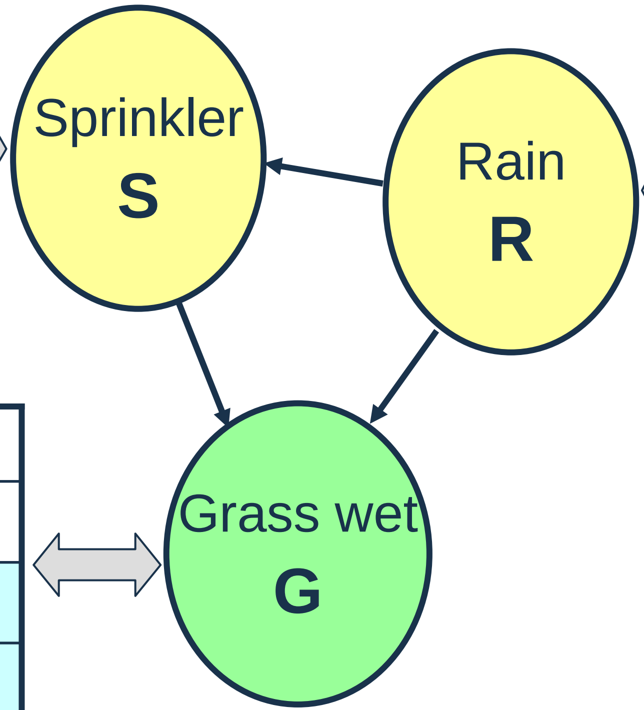


- A Bayesian Network is composed of:
  - A set of variables/events (the "***nodes***")
  - A set of directed edges - or connections – (the "***arrows***") between the variables/events which indicate dependencies, i.e. causal connections.
- The Bayesian Network is a graphical representation of the probabilistic structure of the variables (defined by the "**joint probability density function**" of the variables)
- Each variable/event may have a countable or uncountable set of mutually exclusive states.
- The variables/events together with the directed edges form a ***directed a-cyclic graph*** (DAG)
- To each variable/event with parents B, C, D,.., there is assigned a conditional probability structure P(A/B,C,D,..). The structure is defined by the **CPT** (***Conditional Probability Tables***)
- In case the variable/event A has no parents, the conditional probability structure reduces to the ***unconditional probability*** of A, i.e. P(A)
- The Bayesian Network can provide usual items which refer to probability theory as e.g. expected values, conditional probabilities,…
- **Object-oriented** Bayesian probabilistic network is useful when a phenomenon has many identical probabilistic (sub-) structures.

Bayesian Networks : example

	sprinkler	
rain	T (true)	F (false)
F (false)	0.4	0.6
T (true)	0.01	0.99

CPT table for sprinkler



rain
T (true)	0.2
F (false)	0.8

Unconditional probabilities

rain		F		T	
sprinkler		F	T	F	T
Grass wet	F	1	0.1	0.2	0.01
	T	0	0.9	0.8	0.99

CPT table for grass

Two events could cause grass to be wet:

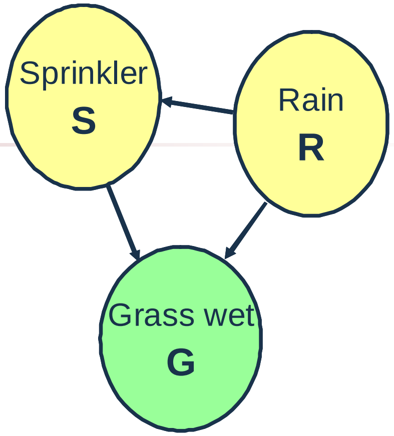
- The sprinkler is on
- It's raining

The rain has a direct effect on the use of the sprinkler (when it rains, the sprinkler is usually not turned on)

The joint probability density function is, according to the "***chain rule***",:

P(G,S,R) = P(G/S,R) x P(S/R) x P(R)

Bayesian Networks : example (continued)



P(G,S,R) = P(G/S,R) x P(S/R) x P(R)

1) **Joint distribution**

P(T,T,T) = 0.2 x 0.01 x 0.99 = 0.00198

P(F,T,T) = 0.2 x 0.01 x 0.01 = 0.00002

P(T,F,T) = 0.2 x 0.99 x 0.80 = 0.15840

P(F,F,T) = 0.2 x 0.99 x 0.20 = 0.03960

P(T,T,F) = 0.8 x 0.40 x 0.90 = 0.28800

P(F,T,F) = 0.8 x 0.40 x 0.10 = 0.03200

P(T,F,F) = 0.8 x 0.60 x 0.00 = 0.00000

P(F,F,F) = 0.8 x 0.60 x 1.00 = 0.48000

$$P(R=T/G=T)=\frac{P(G=T,R=T}{P(G=T)}=\frac{\sum_{S\in[T,F]}P(G=T,S,R=T)}{\sum_{S,R\in[T,F]}P(G=T,S,R)}$$

2) **Marginal distributions**

P(G=F) = 0.55162

P(G=T) = 0.44838

P(R=F) = 0.8

P(R=T) = 0.2

P(S=F) = 0.678

P(S=T) = 0.322

3) **Conditional distributions**

P(R=T/G=T)

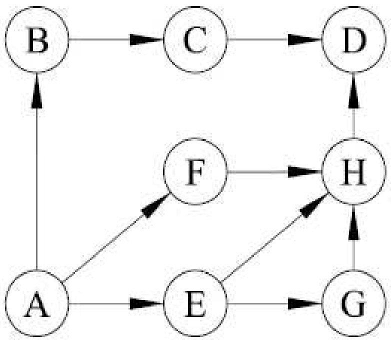
= (0.00198 + 0.1584) / 0.44838

= 0.357688

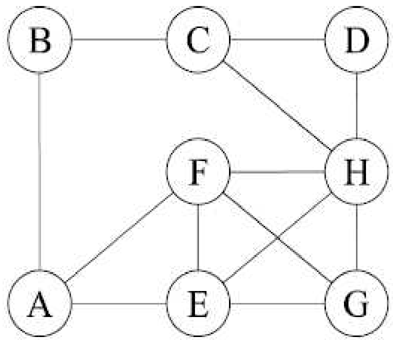
- ***Inference***: conclusion drawn from observations/data.
- ***Bayesian framework allows inference*** using the mathematical rules of probability
- ***Inference engines*** have been developed that makes the calculation more tractable
- The process of construction of the inference engine is called "***compiling***" the model

Bayesian Networks : Inference engines

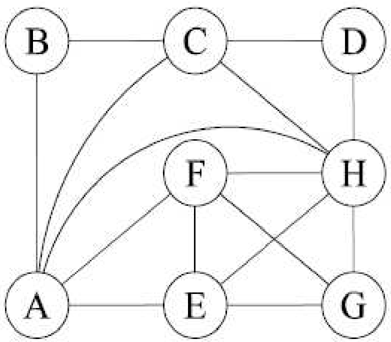
- Manipulating the Bayesian Network structure
- Very efficient so-called **inference engines** have been developed that makes the calculation more tractable than working directly on the initial Bayesian Network



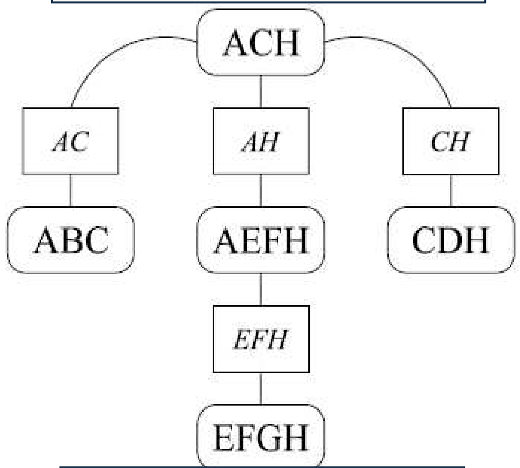
*the **bayesian** network*



*the **moral** network*



*the **triangulated** graph*



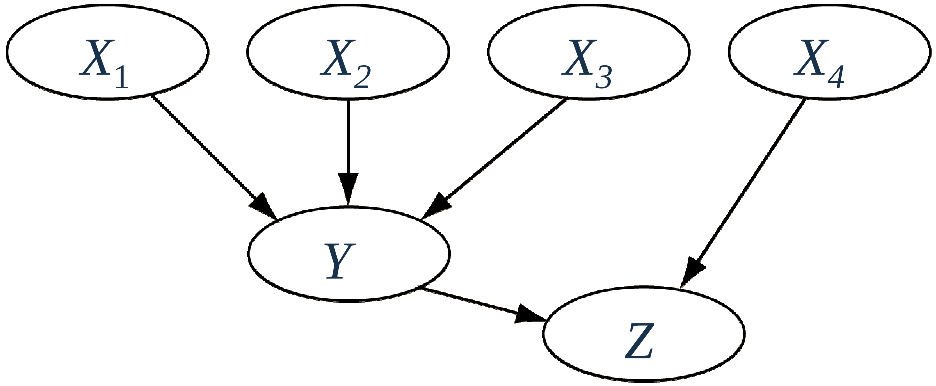
*the **jonction** tree*

- **T**he triangulation phase is in general a very crucial phase as it has a significant influence on the size of the compiled network and thus on the calculation time

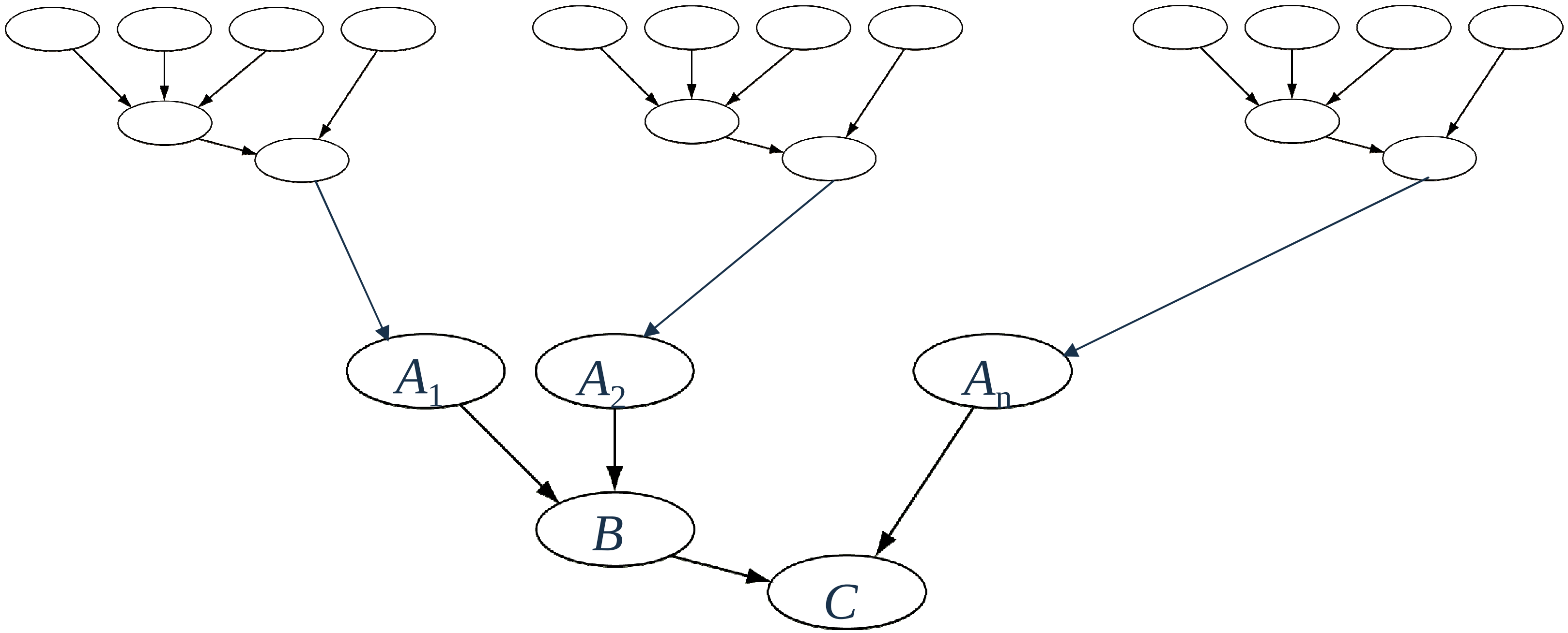
- **R**eferences exist which describe procedures for finding good approximations to the optimal triangulation of a graph

Hierarchical modeling of hull by use of Bayesian probabilistic network

► Component level

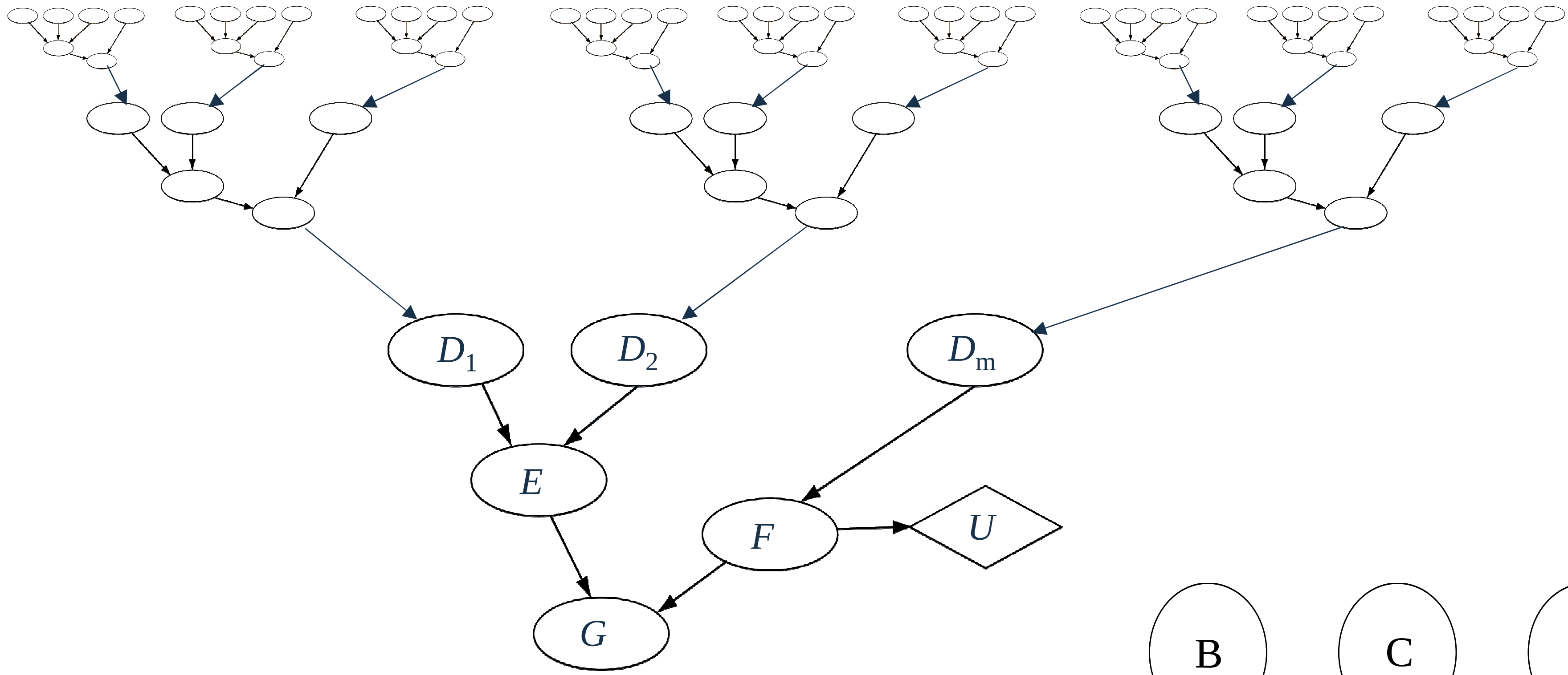


► Sub-structure level

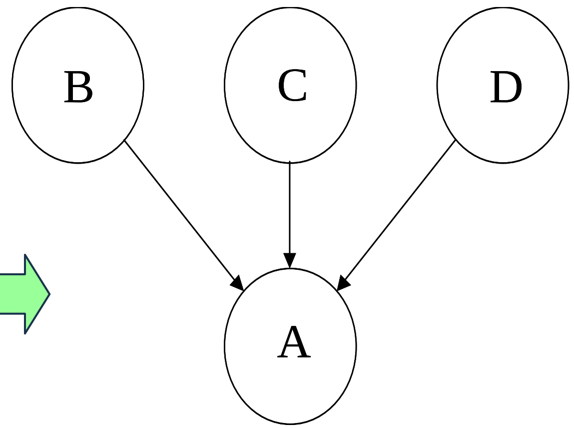


Hierarchical modeling by use of Bayesian probabilistic network

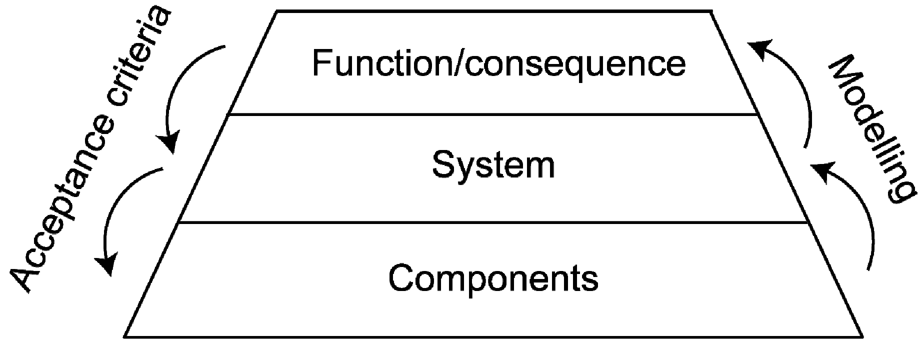
► <u>Structure level</u>



In Bayesian Networks for Hull convergent networks are used



Constrained optimization of components reliabilities in complex systems

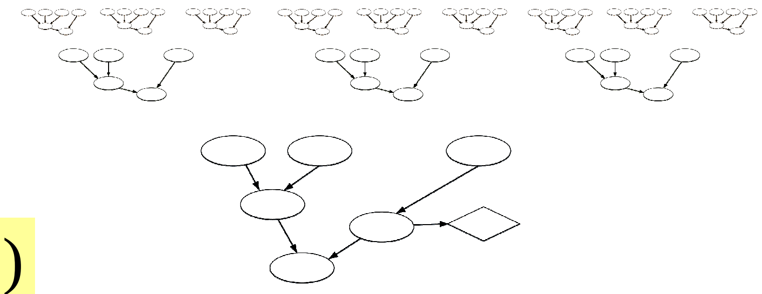


The requirements to the system performance are disaggregated into reliability performance requirements for the components

See "Constrained optimisation of components reliabilities in complex systems", Kazuyoshi Nishijima, Marc A. Maes, Jean Goyet and Michael Havbro Faber, Structural Safety 31 (2009), pages 168-178

Formulation of the optimisation problem

Example 1



On Bayesian probabilistic networks:

expected total cost is written as:

$$u = f(x_1, x_2, ..., x_N)$$

where x_i is design variable for components, e.g. component reliability.

acceptance criteria for system performance are written as:

$$g_j(x_1, x_2, ..., x_N) \leq c_j$$

Optimization of component reliability can be reduced to be a standard constrained optimization problem:

minimize $u = f(x_1, x_2, ..., x_N)$

such that $g_j(x_1, x_2, ..., x_N) \leq c_j \quad ,(j = 1, 2, ..., M)$

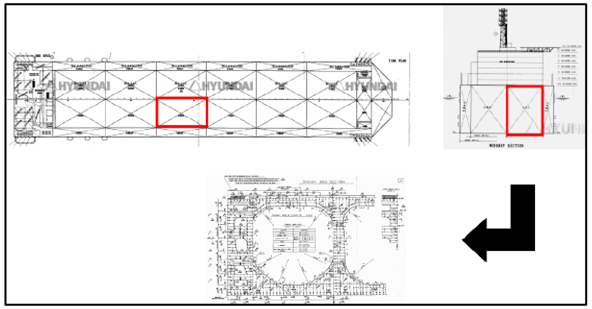
→ solving the optimization problem with commonly available techniques

Hierarchical modelling

Optimization of reliability of welded joints in ship hull structure

Acceptance criterion: *probability of failure of ship hull < 10^{-3}/yr*

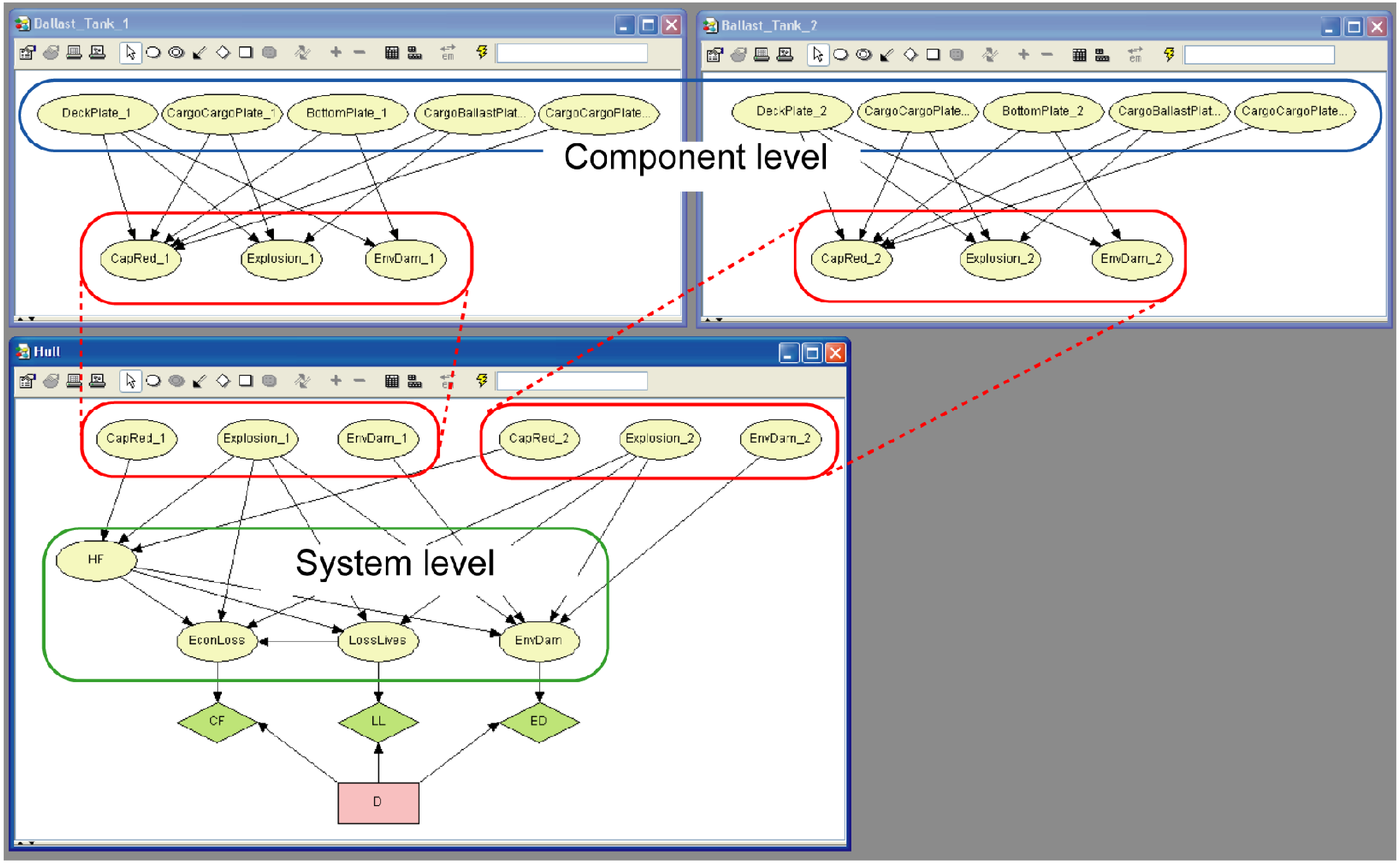
Objective function : *expected total cost*



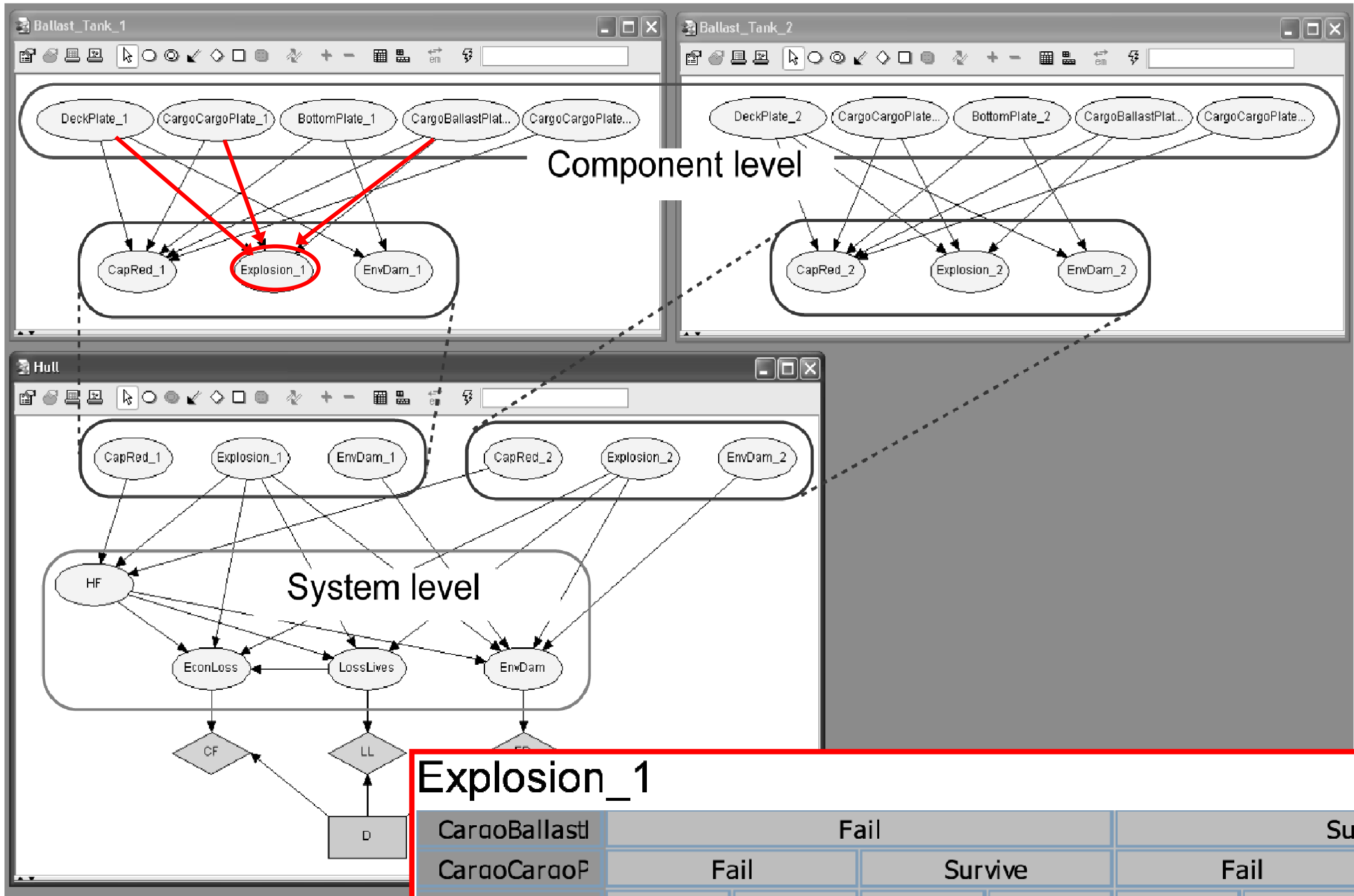
Hierarchical structure of the ship hull:

Hull structure	Tanks	Components
Hull	Ballast tank 1	Deck plate
	Ballast tank 2	Bottom plate
	⋮	Side plate
	Cargo tank 1	Tank partition
	⋮	⋮

Corresponding BPN's:



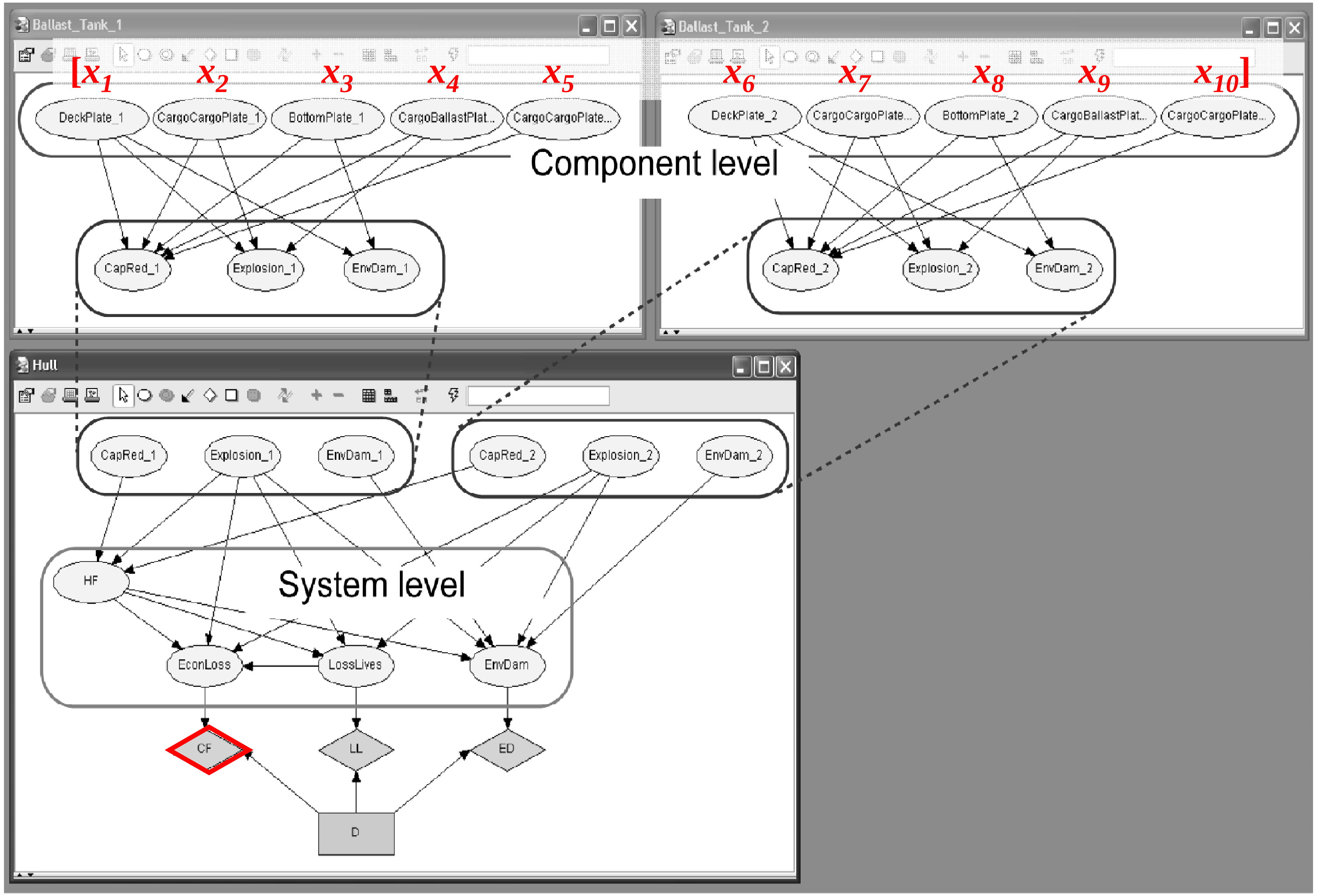
Conditional probability tables



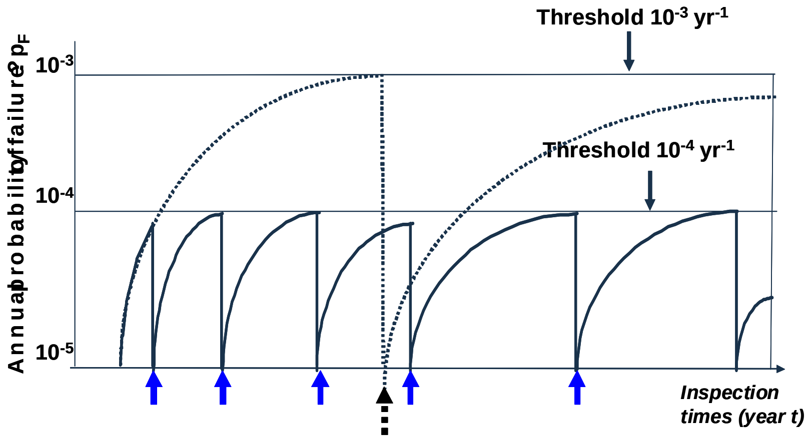
Explosion_1

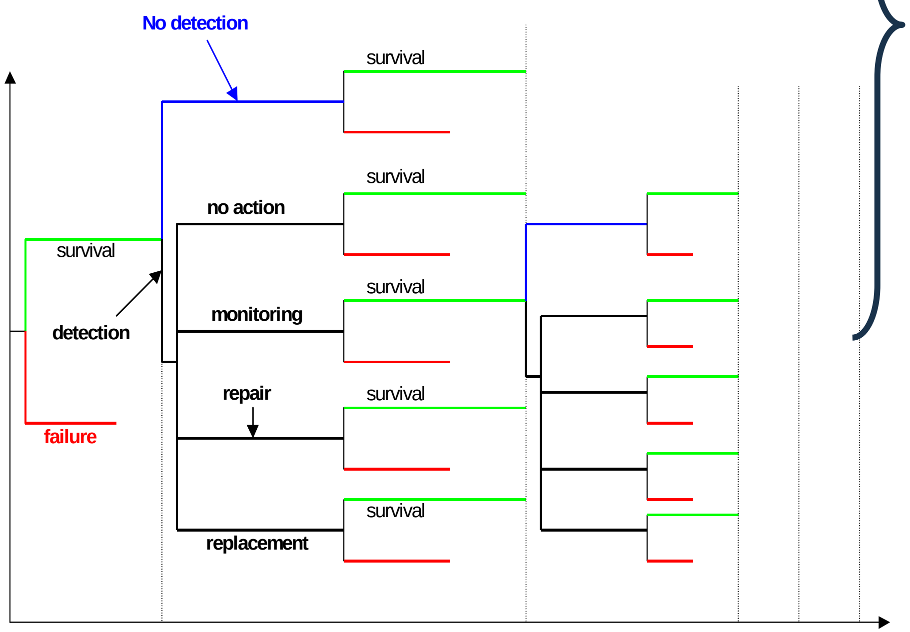
CargoBallastl	Fail				Survive			
CargoCargoP	Fail		Survive		Fail		Survive	
DeckPlate_1	Fail	Survive	Fail	Survive	Fail	Survive	Fail	Survive
No	0.4	0.5	0.8	0.8	0.4	0.5	0.6	1
Minor	0.5	0.45	0.1	0.19	0.55	0.49	0.35	0
Major	0.1	0.05	0.1	0.01	0.05	0.01	0.05	0

Objective function: $u = f(x_1, x_2, \ldots, x_{10})$

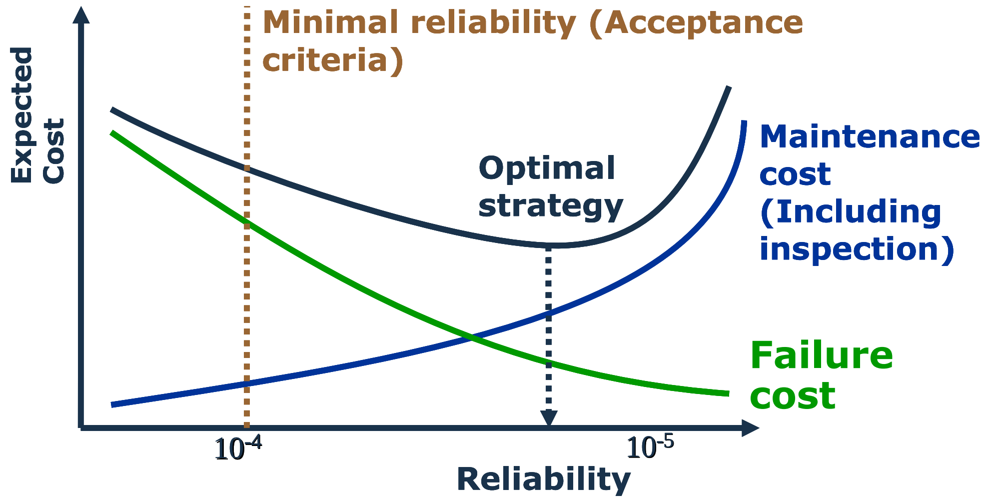


RBI Hull – Quantitative Methodology and cost calculation Example 1

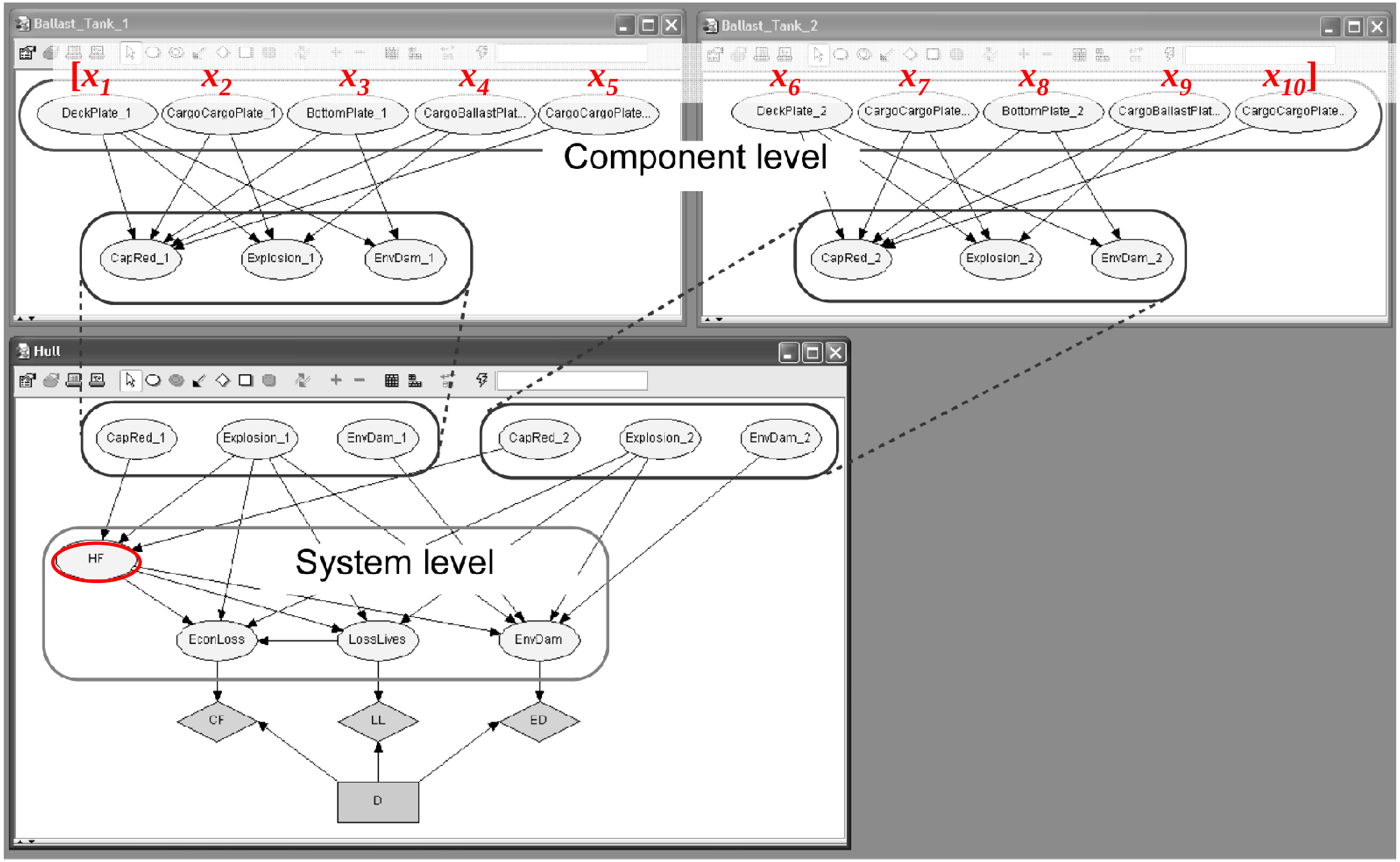




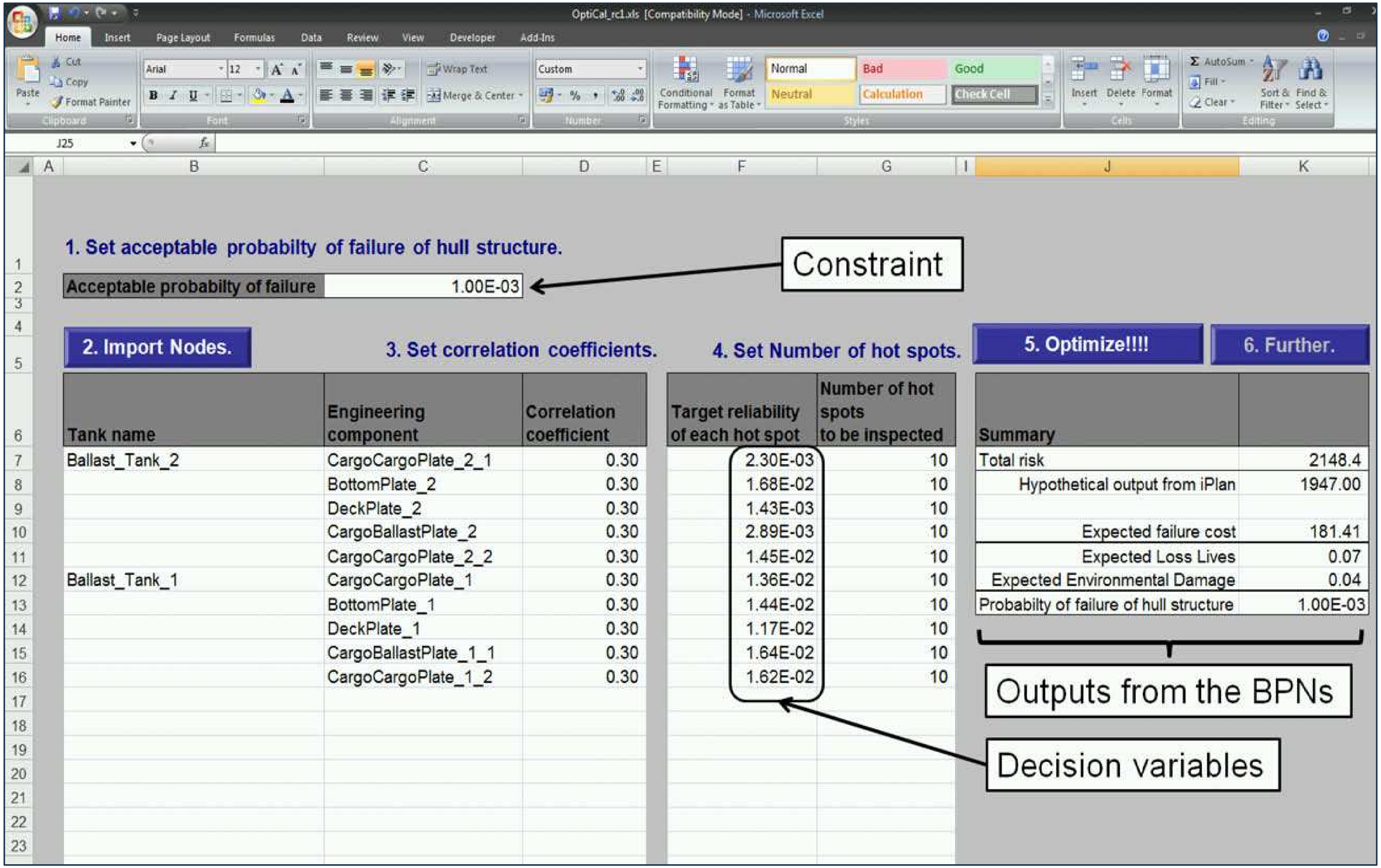
- minimize $E[C_T] = \min \Sigma C(S_i) P(S_i)$



Constraints: $g(x_1, x_2, \ldots, x_{10}) \le c$



Results

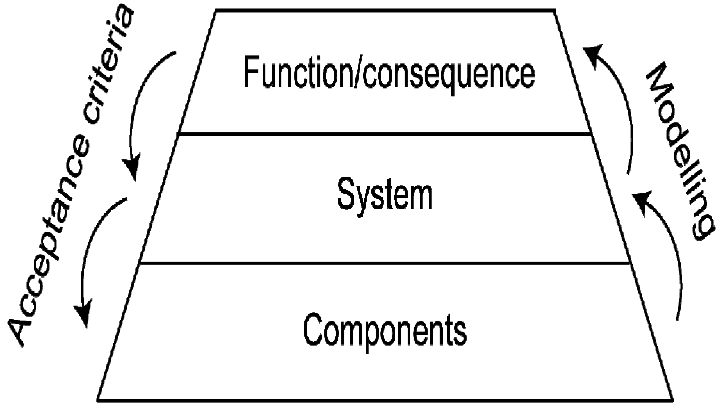


Acceptable probabilty of failure	1.00E-03

Tank name	Engineering component	Correlation coefficient	Target reliability of each hot spot	Number of hot spots to be inspected
Ballast_Tank_2	CargoCargoPlate_2_1	0.30	2.30E-03	10
	BottomPlate_2	0.30	1.68E-02	10
	DeckPlate_2	0.30	1.43E-03	10
	CargoBallastPlate_2	0.30	2.89E-03	10
	CargoCargoPlate_2_2	0.30	1.45E-02	10
Ballast_Tank_1	CargoCargoPlate_1	0.30	1.36E-02	10
	BottomPlate_1	0.30	1.44E-02	10
	DeckPlate_1	0.30	1.17E-02	10
	CargoBallastPlate_1_1	0.30	1.64E-02	10
	CargoCargoPlate_1_2	0.30	1.62E-02	10

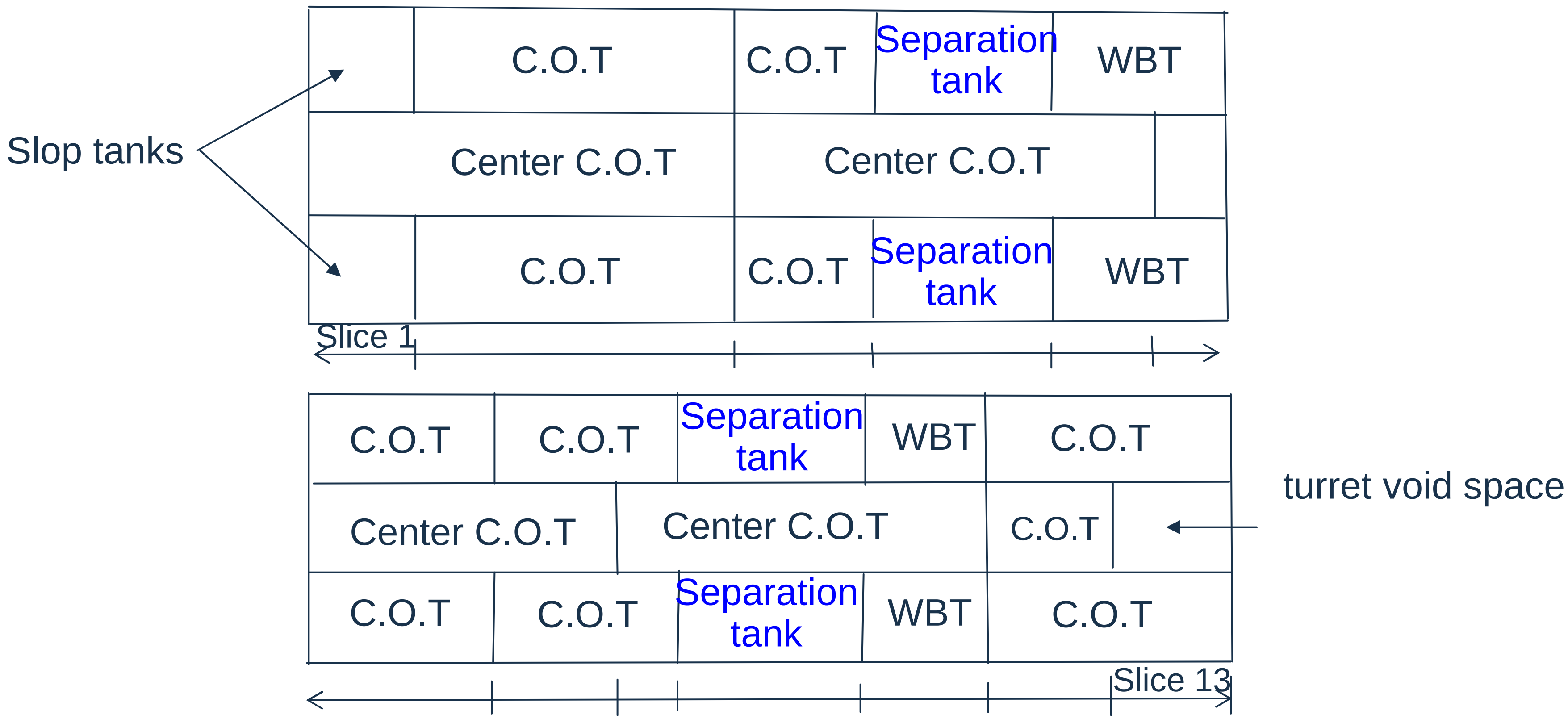
Summary	
Total risk	2148.4
Hypothetical output from iPlan	1947.00
Expected failure cost	181.41
Expected Loss Lives	0.07
Expected Environmental Damage	0.04
Probabilty of failure of hull structure	1.00E-03

Checks of performances at system level with unconstrained components reliabilities



See: “Risk Assessment in Engineering – Principles, System Representation & Risk Criteria, Annex, Example : Risk Based Inspection of Offshore Structures”, JCSS (Joint Committee on Structural Safety), J. Goyet, Antoine Rouhan and Fernando Castanheira (Bureau Veritas), Bruno Farias (Petrobras), Michael Faber and Kazuyoshi Nishijima (ETH), 15th of April 2010, *may be downloaded on the JCSS Website*

Example : Risk Based Inspection of FPSO Hull structure



- ► Risk Analysis may be used in inspection planning
  - ❑ either as alternative to usual prescriptive rules
  - ❑ or as complement to these rules
- ► Risk Analysis is performed using
  - ❑ a **hierarchical model** of the hull
  - ❑ **Bayesian Networks** fitted to the hierarchical model

RBI approach at system level - steps

- ***Step 1***: **Risk Acceptance Criteria**

- ***Step 2***: **Cargo area subdivision and definition of inspection plans**
  - Inspection plans are pre-defined for the unit, for example a set of inspection times T_1, T_2, T_3 are established for each type of tanks (ballasts, crude oil tanks,..)

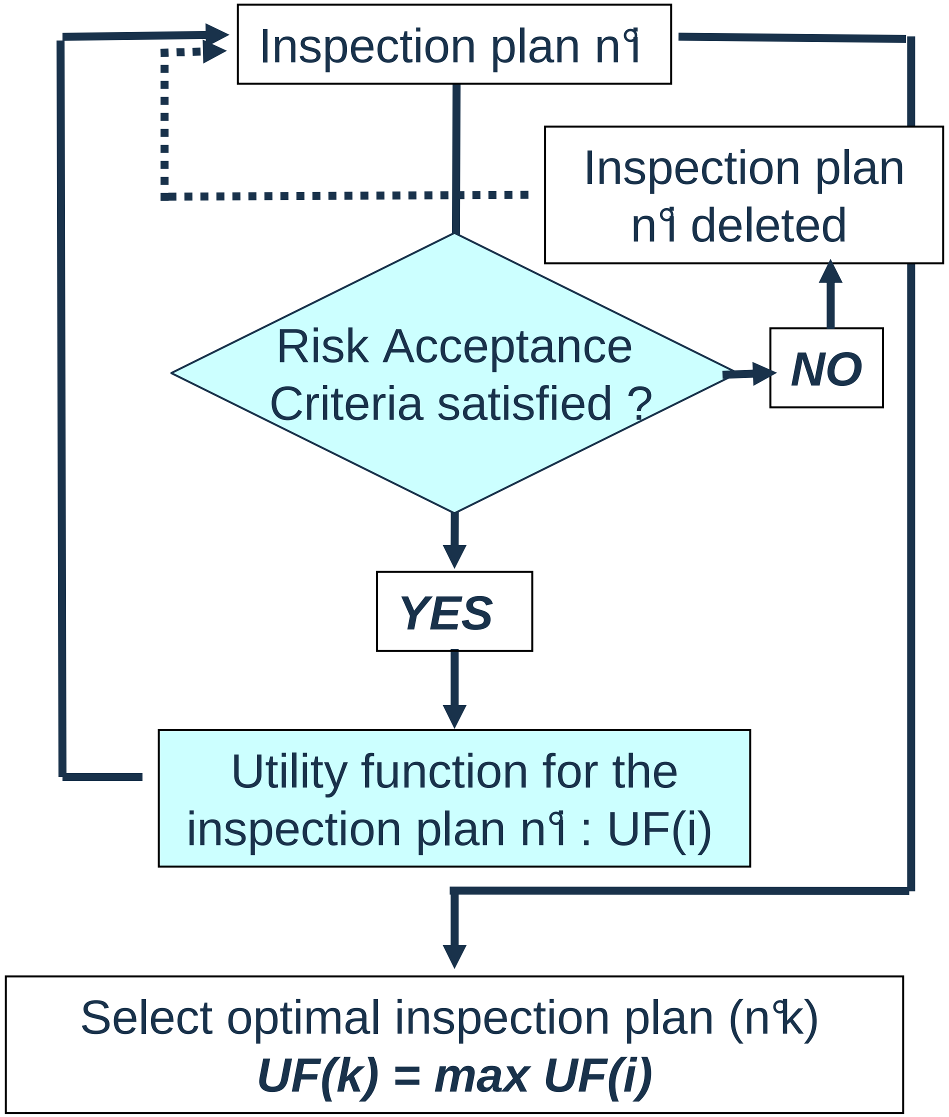
- ***Step 3***: **Annual damage state of the unit taking into account:**
  - degradation mechanisms (general corrosion, pitting, fatigue)
  - inspection planning
  - mitigation strategy

- ***Step 4***: **Risk Analysis of the unit on an annual basis:**
  - using damage states determined in step 3
  - using **B**ayesian **P**robabilistic **N**etworks (BPN) for structural and explosion analyses
  - taking into account all transverse sections of the unit

- ***Step 5:*** ***Check*** **risk acceptance criteria annually for Personnel and Environment**

- ***Step 5***: **Optimisation (Economical criteria)**
  - Optimisation is performed over the service life (summation over the years)
  - Alternative inspection plans are compared (the optimal one is selected)

Step 5: Optimisation (Economic criteria)



- The utility function UF in this example is the total expected cost associated to a given inspection plan.
- This total expected cost includes:
  - o the ***cost of inspection***,
  - o the ***cost of repair***
  - o and the ***cost of failure***

Risk checking at system level : Principle of the BPN

The Bayesian Probabilistic Network (BPN) is intended to:

- Calculate the consequences of structural component failures in terms of loss of lives, environment and economics.

- Check the Risk Acceptance Criteria (loss of lives, environment) and

- Calculate the total expected cost associated to each inspection plan and select the optimal one.

The hull is modelled using a hierarchical model. This hierarchical model allows for determining **event scenarios** which start at the lowest level (initial event, defined at component level) and go trough the hierarchical model up to the highest level (terminal event, dealing with the hull as a whole which is analysed in terms of final consequences: loss of lives, environment and economics).

The BPN performs its calculations via the hierarchical model. Input data are:

o the hierarchical model itself, which takes into account naval architecture and hull constructive aspects

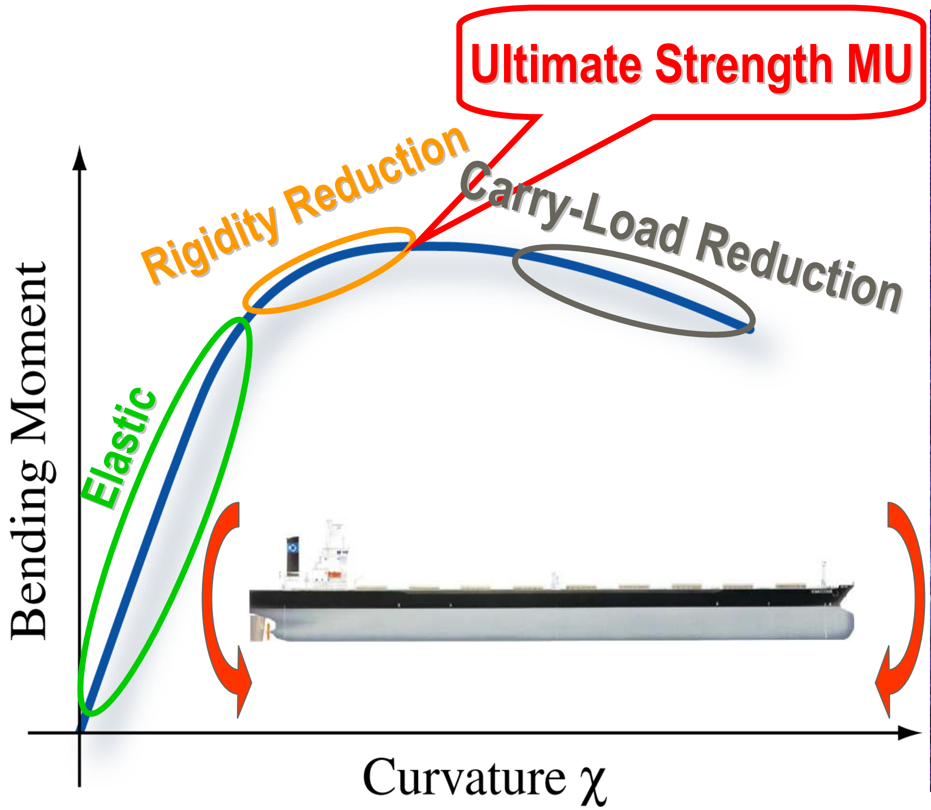
o pure and conditional probabilities along event scenarios

o consequences associated with terminal events (loss of lives, environment and economics).

:Therefore, three steps are included in the BPN construction: Event Scenarios Analysis, definition of the Hierarchical Model and construction of the BPN itself, i.e. to join the hierarchical model with all involved probabilities (pure and conditional) and final consequences.

$$1.0\ M_{sw} + 1.20\ M_{wv} \leq MU / 1.10$$

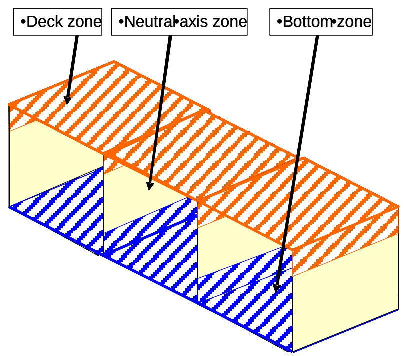
Permissible SWBM

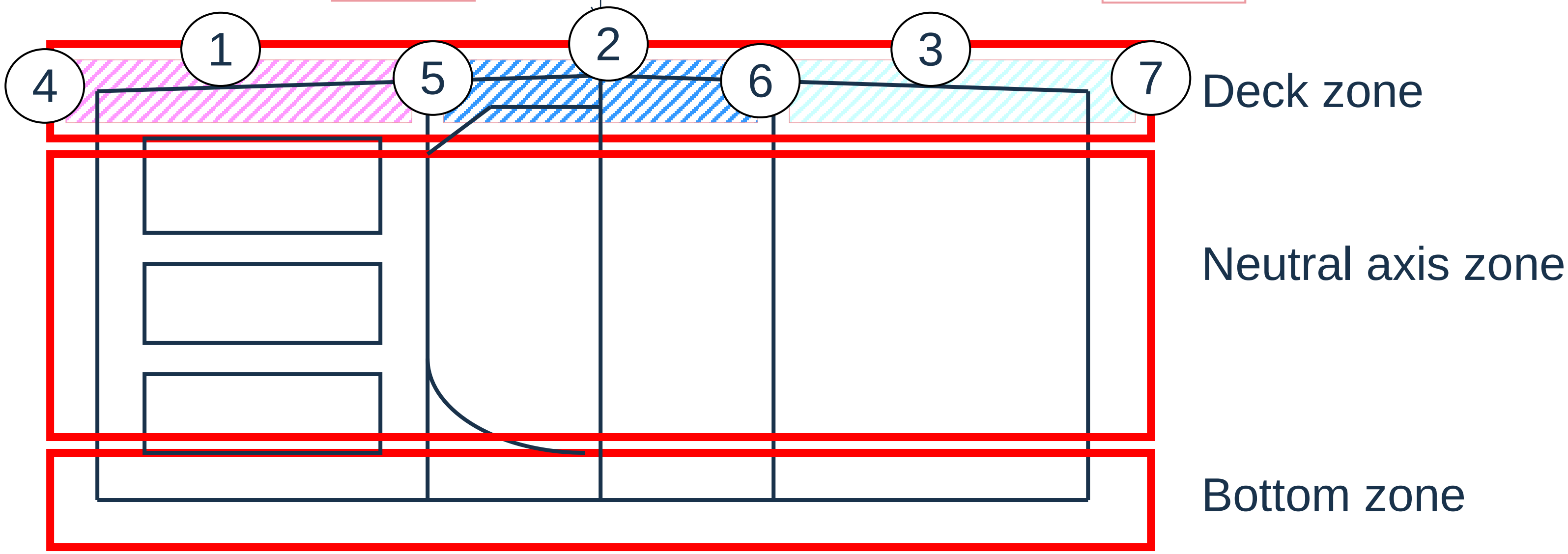


Hierarchical model for hull (cargo region) - 1 -

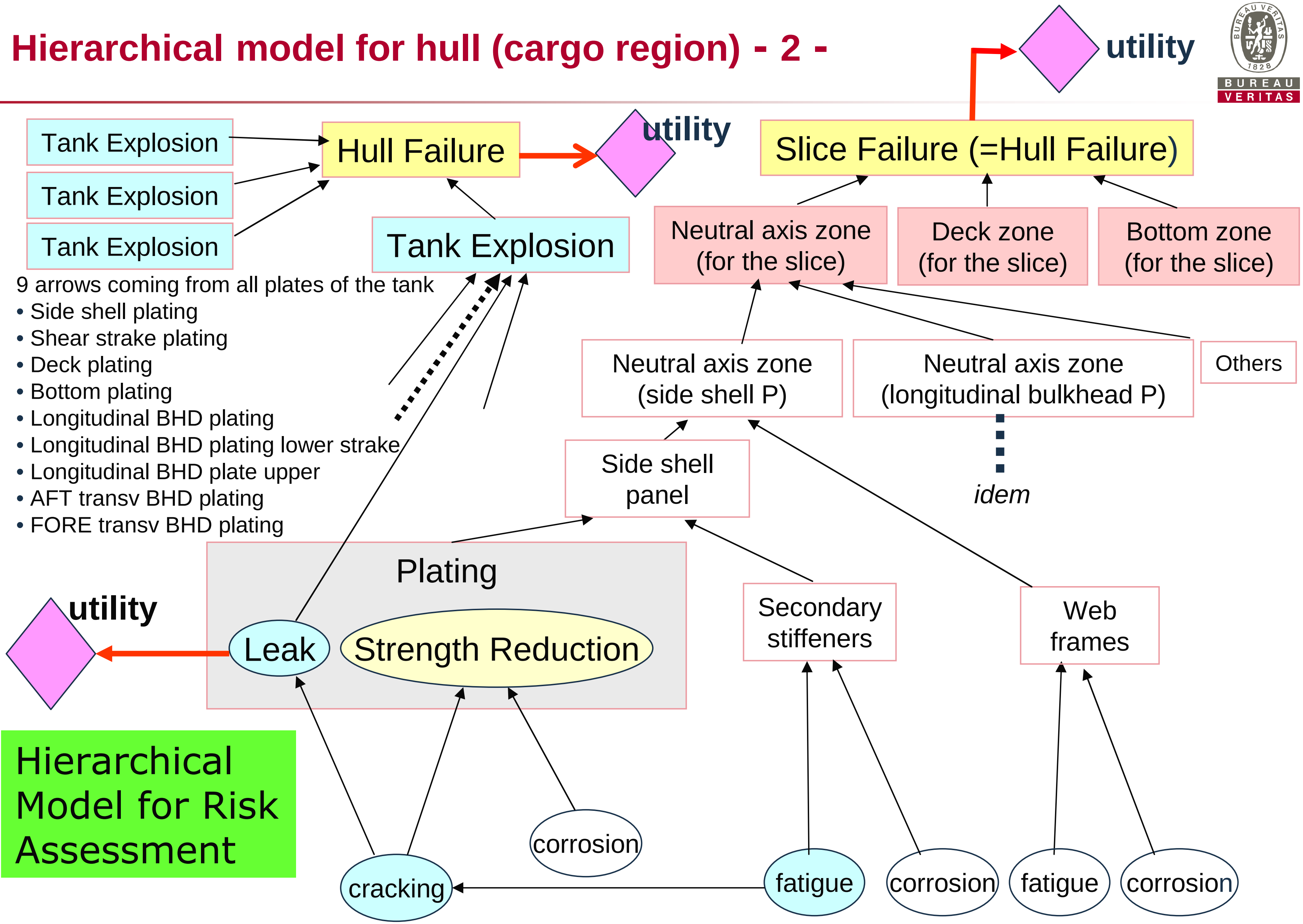
Deck Zone for transversal section

- Deck zone (wing tank)
  - Deck panel
    - Plating
    - Secondary stiffeners
  - Web frames
- Deck zone (center tank)
- Deck zone (wing tank)
  - Deck panel
    - Plating
    - Secondary stiffeners
  - Web frames



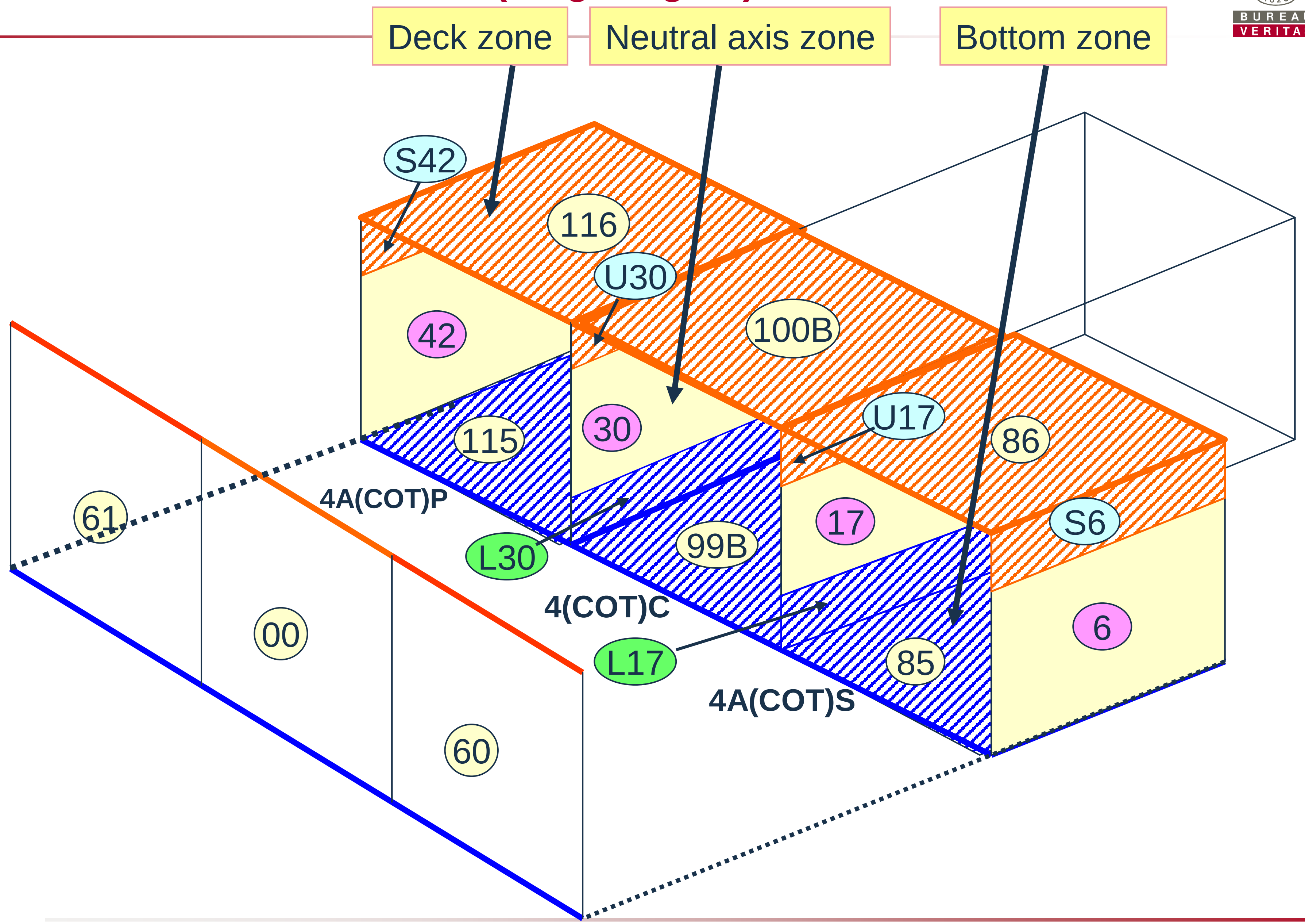


Hierarchical model for hull (cargo region) - 2 -

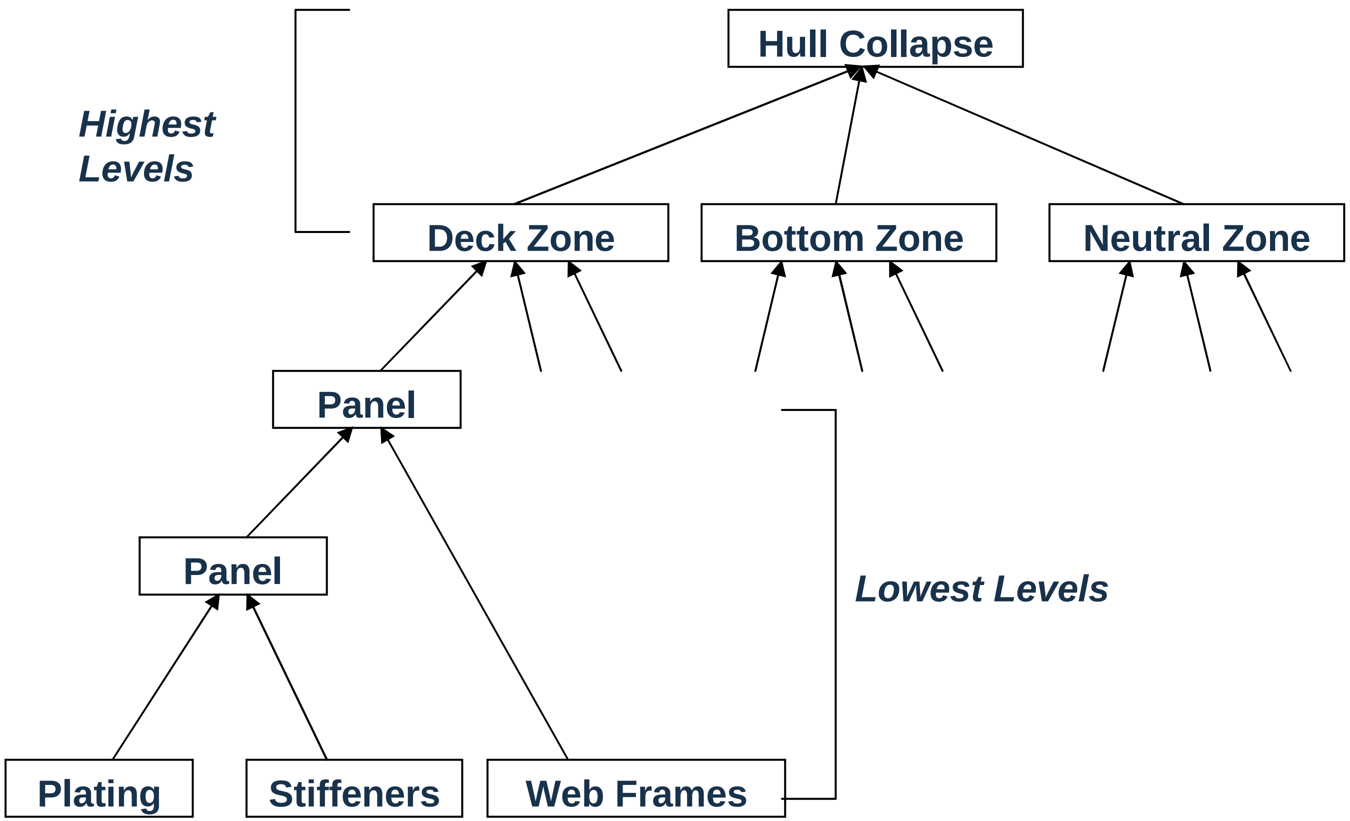


Hierarchical Model for Risk Assessment

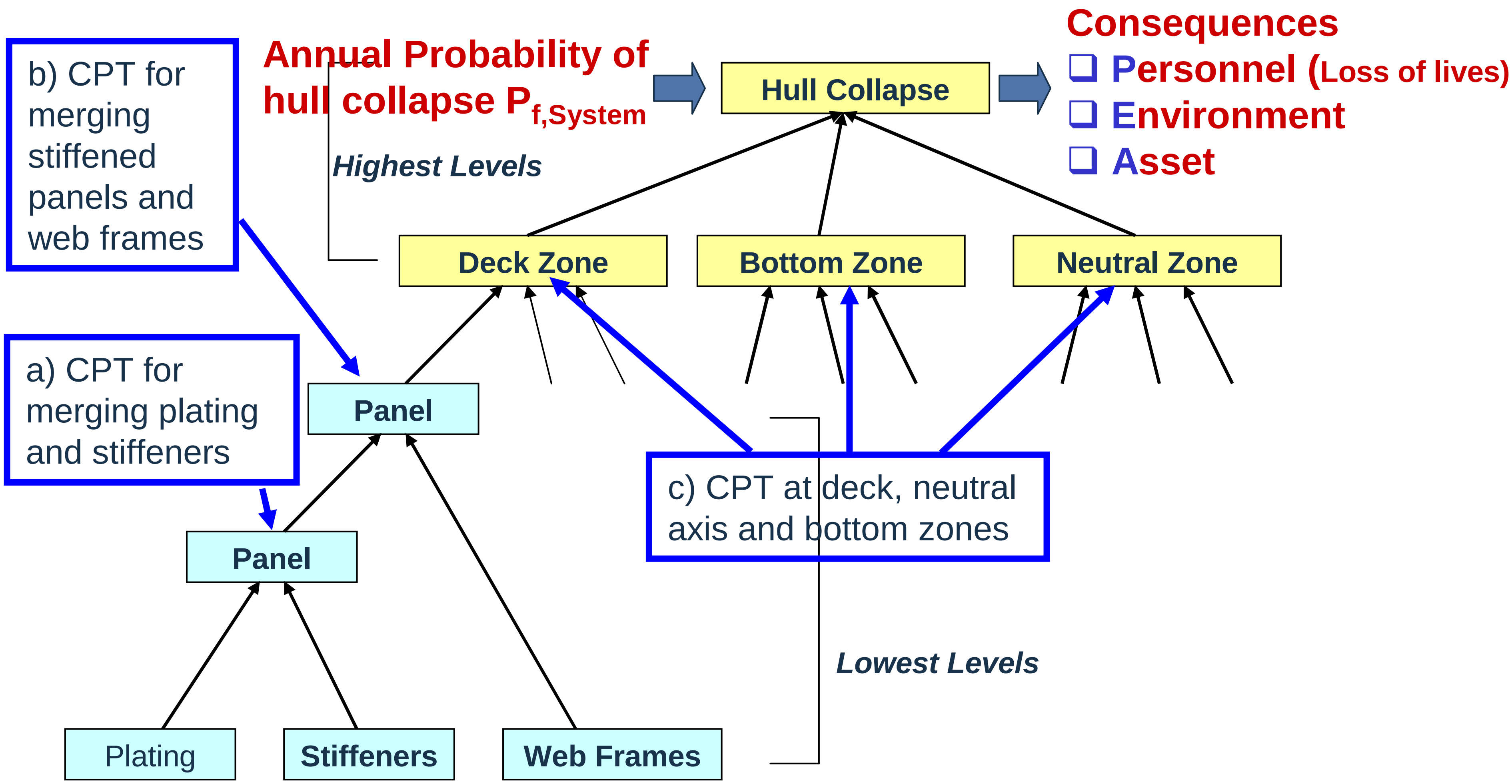
Hierarchical model for hull (cargo region) – 3 -



Hierarchical model for hull (cargo region) - 4 -



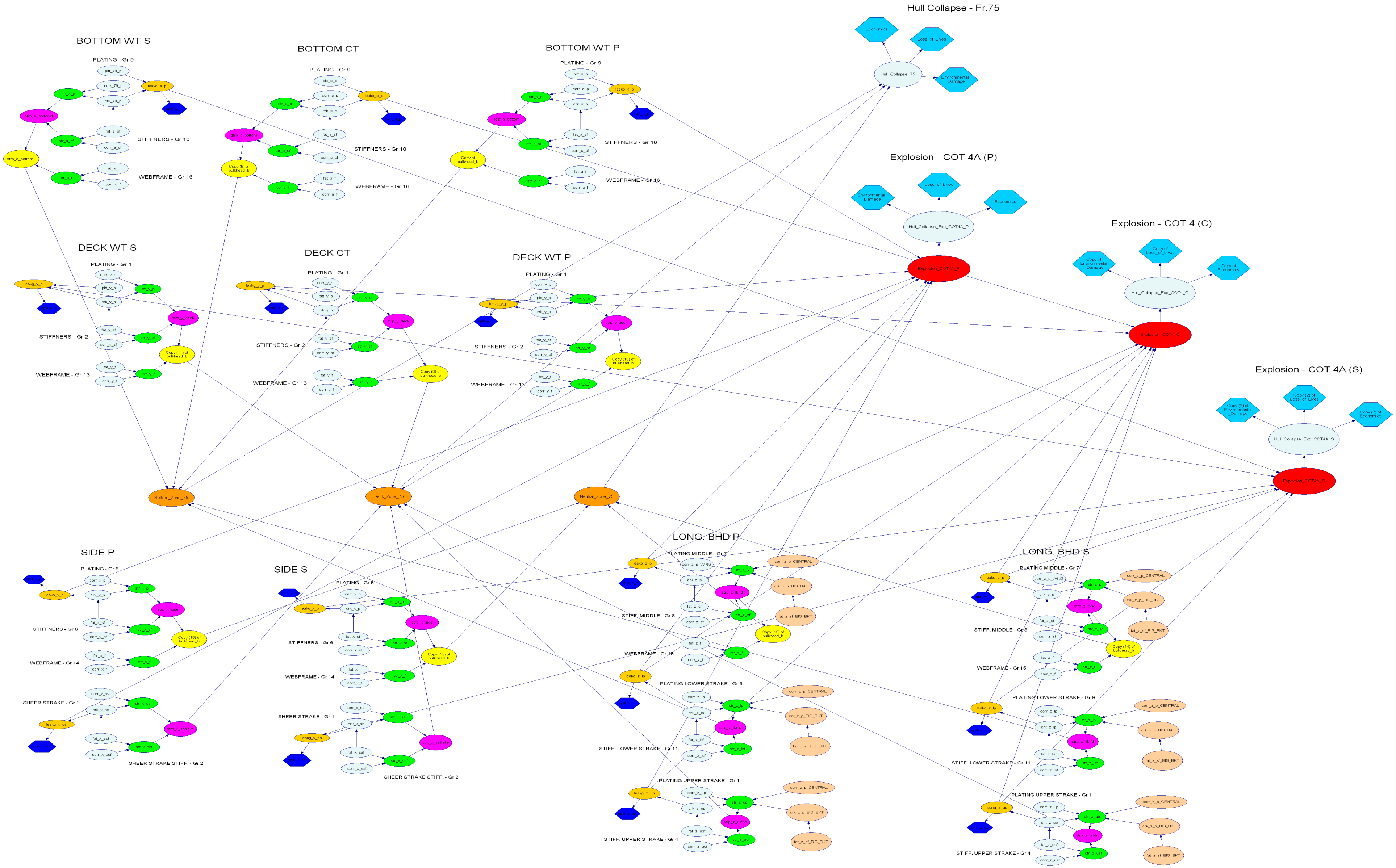
Hierarchical model for hull (cargo region) - 5 -



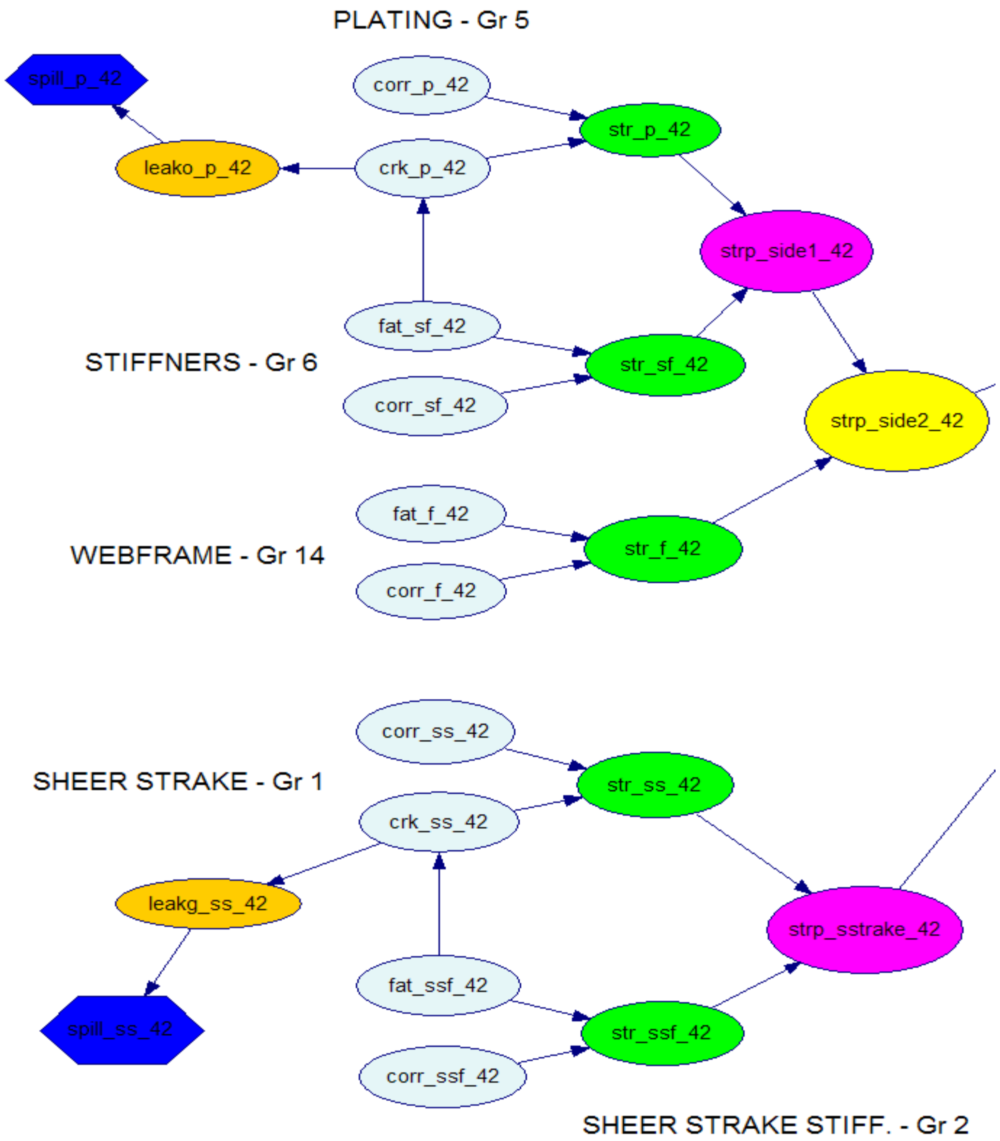
CPT : Conditional Probability Tables derived from expert knowledge or FEM calculations

	Gives the Conditional Probabilities to Have:	Due to:
	Cracking in the plating (through cracks)	Fatigue in the stiffeners
Highest level	Strength reduction of plating	Cracking and/or general corrosion
	Strength reduction of stiffeners	Fatigue and/or general corrosion
	Strength reduction of bottom frames	Fatigue and/or general corrosion
Intermediate level	Strength reduction of reinforced panel (plating + stiffeners)	Strength reduction of plating and/or stiffeners
	Strength reduction of bottom reinforced panel (plating + stiffeners + frames)	Strength reduction of bottom reinforced panel (plating + stiffeners) and/or frames
	Strength reduction of the structural zones (bottom, neutral and deck) for each hull slice (total of 13 slices)	Strength reduction of the reinforced panels within each zone
Lowest level	Hull Collapse	Strength reduction of the structural zones (bottom, neutral and/or deck) for each hull slice (total of 13 slices)
	Leakage through the plating	Pitting and/or cracking in the plating

The complete BPN for a particular slice of the hull



lowest level of the BPN (Side shell)



Formulation

Annual Risk assessment

For each year of the service life (i=1, NA), the process is as follows:

run the Structural integrity BPN slice per slice (see figure 1 for the definition of slices)

run the Explosion BPN

Items of importance are:

The expected utility for Personnel

$$U_P(i) = \sum_{j=1}^{13} E_{HF}(P, j) + \sum_{j=1}^{13} E_{EXP/TR}(P, j) + \sum_{l=1}^{26} E_{EXP/TK}(P, l) \qquad (1)$$

The expected utility for environment

$$U_E(i) = \sum_{j=1}^{13} E_{HF}(E, j) + \sum_{j=1}^{13} E_{EXP/TR}(E, j) + \sum_{l=1}^{26} E_{EXP/TK}(E, l) + \sum_{j=1}^{13}\sum_{k=1}^{19} E_{LEAK}(E, j, k) \qquad (2)$$

The expected cost for economics

$$U_A(i) = \sum_{j=1}^{13} E_{HF}(A, j) + \sum_{j=1}^{13} E_{EXP/TR}(A, j) + \sum_{l=1}^{26} E_{EXP/TK}(A, l) + \sum_{j=1}^{13}\sum_{k=1}^{19} E_{LEAK}(A, j, k) \qquad (3)$$

Formulation (continued)

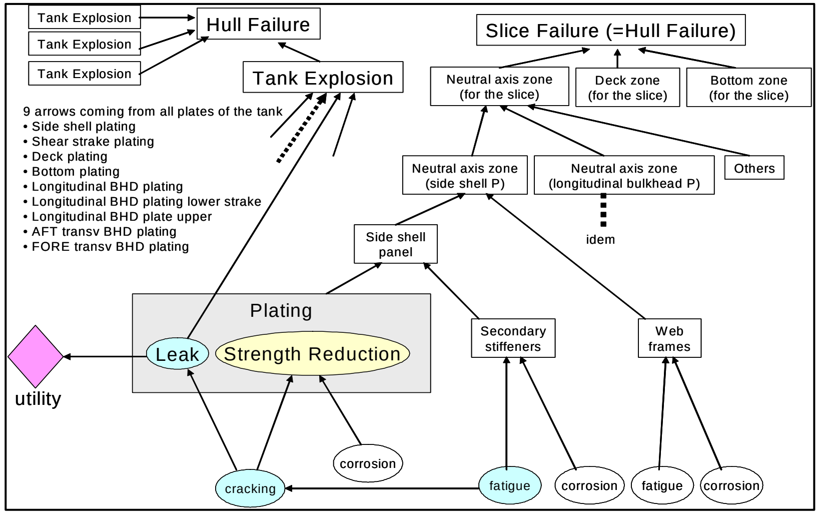
Checking of acceptance criteria (on an annual basis):

For personnel

$U_P(i) < RAC_{P,annual}$ / i=1, NA

Pour environment

$U_E(i) < RAC_{E,annual}$ / i=1, NA



Optimisation

Utility function for economics is calculated for each alternative inspection plan by summation over the service life:

$$U_A = \sum_{i=1}^{N_A} U_A(i) \qquad (4)$$

Utility for economics in (4) has now to include cost of inspection and cost of repair. So (3) is written as:

$$U_A(i) = \sum_{j=1}^{13} E_{HF}(A,j) + \sum_{j=1}^{13} E_{EXP/TR}(A,j) + \sum_{l=1}^{26} E_{EXP/TK}(A,l) + \sum_{j=1}^{13}\sum_{k=1}^{19} E_{LEAK}(A,j,k) + E(C_I) + E(C_R) \qquad (5)$$

The optimal plan is the plan which minimises the utility function UA(i):

$$U_A^O = \min\left(U_A^m\right),\ m = 1, NI \qquad (6)$$

Where NI is the number of inspection plans under investigation.

Analysis and results

► **Step 1** - **Risk Acceptance criteria at system level** are derived from Risk Matrices from the owner (for personnel it was set to 8 x 10^{-5} loss of lives per year)

► **Step 2** - **Hull area subdivision and definition of inspection plans**

- 4 groups of tanks: ballast tanks, separation tanks, center tanks, other wing tanks

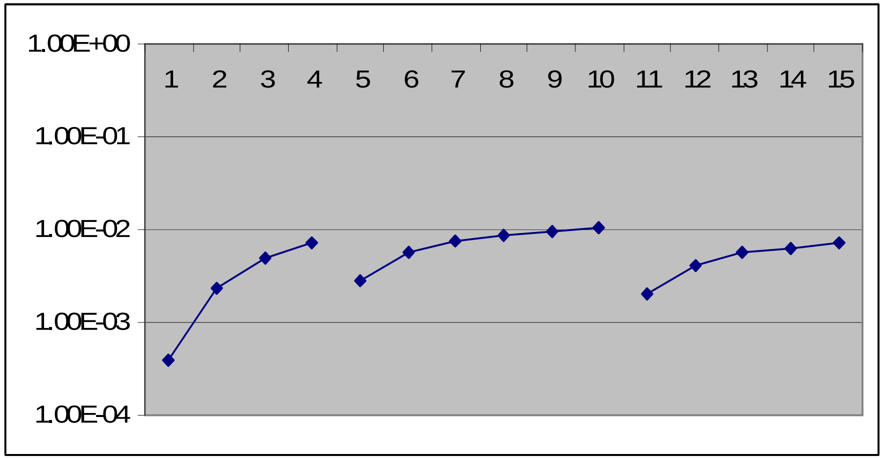
- 5 types of inspection plans (given below in terms of years of inspection)

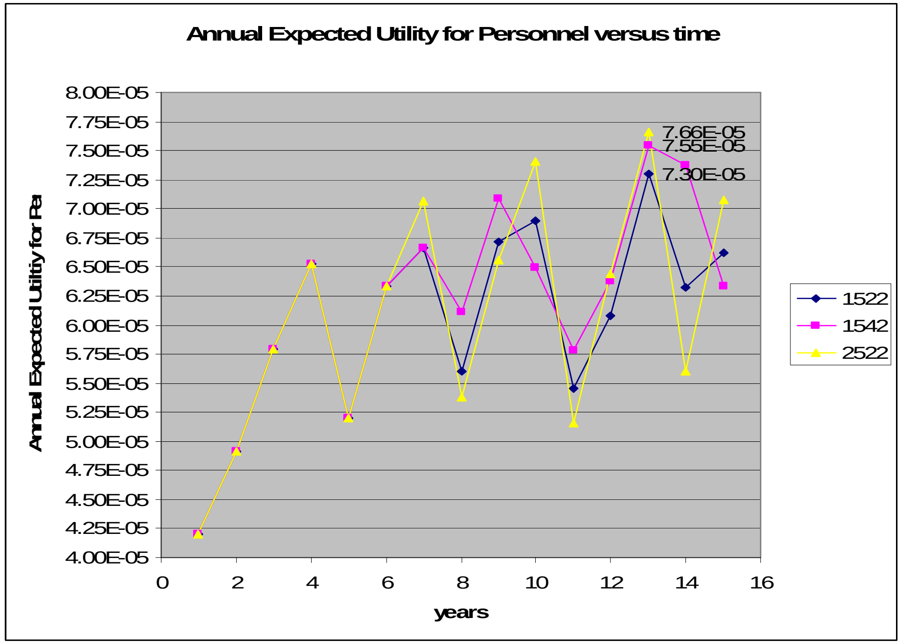
  - *type 1*: **4**, **6**, **9**, **11**, **14**
  - *type 2*: **4**, **7**, **10**, **13**
  - *type 3*: **4**, **8**, **13**
  - *type 4*: **4**, **9**, **14**
  - *type 5*: **4**, **10**

- Each group of tank may have one of the 5 types of inspection: **625** potential inspection plans

Analysis and results (continued)

- **Step 3** – **Annual damage state of the unit** taking into account
  - Degradation mechanisms (fatigue, general corrosion, pitting)
  - Inspection plan
  - Surveys findings / mitigation strategy
  - Input data (in the BPN's) for the annual damage state: 13 x 107 = 1391 components (probabilities of failure)
- **Step 4** - **Risk Analysis of the unit on an annual basis**
  - Risk Analysis performed for the 625 potential inspection plans
- **Step 5** – **Optimisation** according to formulation





Thank you for your attention